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Advanced Linguistic Complex T-Spherical Fuzzy Dombi-Weighted Power-Partitioned Heronian Mean Operator and Its Application for Emergency Information Quality Assessment

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Abstract: Against the background of a major change in the world unseen in a century, emergencies with high complexity and uncertainty have had serious impacts on economic security and sustainable social development, making emergency management an important issue that needs to be urgently resolved, and the quality assessment of emergency information is a key link in emergency management. To effectively deal with the uncertainty of emergency information quality assessment, a new fuzzy multi-attribute assessment method is proposed in this paper. First, we propose the linguistic complex T-spherical fuzzy set (LCT-SFS), which can deal with two-dimensional problems and cope with situations in which assessment experts cannot give quantitative assessments. Then, the advanced linguistic complex T-spherical fuzzy Dombi-weighted power-partitioned Heronian mean (ALCT-SFDWPPHM) operator, which incorporates the flexibility of Dombi operations, is proposed. The partitioned Heronian mean (PHM) operator can consider attribute partitioning and attribute correlation, the power average (PA) operator can eliminate the effect of evaluation singularities, and the advanced operator can circumvent the problem of consistent or indistinguishable aggregation results, which provides a strong comprehensive advantage in the evaluating information aggregation. Finally, a fuzzy multi-attribute assessment model is constructed by combining the proposed operator with the WASPAS method and applied to the problem of assessing the quality and sensitivity of emergency information; qualitative and quantitative comparison analyses are carried out. The results show the method proposed in this paper has strong feasibility and validity and can represent uncertainty assessment more flexibly while providing reasonable and reliable results. The method can provide new ideas and methods for the quality assessment of emergency information, and promoting sustainable, efficient, and high-quality development of emergency management.

Keywords: linguistic complex T-spherical fuzzy set; Dombi-weighted power-partitioned Heronian mean operator; sustainable development; the quality assessment of emergency information; multi-attribute assessment



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1. Introduction

Emergencies have been occurring frequently in recent years, causing emergency management to enter the normalization stage due to their suddenness, uncertainty, dynamicity, and derivation characteristics. High-quality emergency information can help us better identify the risks faced by social, environmental, and economic systems to develop corresponding emergency plans. It can increase the effectiveness of emergency management in achieving sustainability in social development [1,2]. With the increasing degree of social informatization and networking and the large amount of data emerging shortly after emergencies, the effective screening of emergency information has become an urgent problem to be solved. The high-quality emergency information obtained through filtering can be

applied more effectively to handle emergency events and further improve the efficiency of emergency management [3]. As a key link in filtering emergency information, emergency information quality assessment has become a crucial factor in improving emergency management capability and implementing sustainable development strategies. Some scholars have conducted studies related to the quality assessment of emergency information. In terms of the characteristics of emergency information quality, Lamb et al. [4] stated that high-quality emergency management information should be characterized by accuracy and reliability. Seppänen and Virrantaus [5] argued that in addition to accuracy characteristics, timeliness is an important reflection of the quality of emergency information. Aggarwal [6] further enriched the study of emergency information quality by stating that information accuracy, timeliness, relevance, and consistency are important manifestations. Kauphold et al. [7] analyzed the quality of emergency information in social media during crises from a practical point of view and found that it is also characterized by reliability, comprehensibility, and timeliness. Based on the analysis of the characteristics of emergency information, Liu et al. [8] constructed a complete set of emergency information quality index systems from the three core dimensions of information content, expression, and utility. Some scholars propose TOPSIS [9] and VIKOR [10] assessment methods for the quality management of emergency information from a methodological perspective to better serve emergency management practice.

The above research has improved the reasonableness of the quality assessment results of emergency information, leading to a gradual optimization of the results. However, current research on the theory of the quality assessment of emergency information remains weak, and no work has focused on the quality assessment of emergency information under uncertainty as the complexity of the assessment environment increases. Therefore, the theory of the quality assessment of emergency information needs to be further improved to meet the increasing demand for the quality assessment of emergency information. The quality assessment of emergency information is a complex, uncertain, and multi-dimensional systematic project [11] that is neither the independent influence of a single attribute nor the simple sum of multiple attributes and exhibits the complex characteristics of multi-structure, multi-type, multi-objective, and multi-factors [12]. Thus, the quality assessment of emergency information should be analyzed from the perspectives of multi-dimensions and multi-attributes and the overall joint effect of the interrelated influences of multiple attributes on the quality assessment of emergency information should also be assessed. Therefore, this paper focuses on introducing advanced and scientific multi-attribute assessment methods into quality assessment of emergency information and exploring the best way to construct a quality assessment model of emergency information.

Zadeh [13] proposed the fuzzy set theory to deal with the uncertainty of preference information in the assessment process. With the deepening of the complexity of the evaluation problem, Atanassov [14] defined the intuitionistic fuzzy set (IFS) that can consider the membership and non-membership; that is, the sum of membership and non-membership should be between 0 and 1. Yager successively introduced the Pythagorean fuzzy set [15] (PyFS) and q-rung orthopair fuzzy set [16] (qROFS), which are more representative than IFS, and effectively broadened the description dimension of uncertain information, which have more flexibility and applicability in the decision-making process. The common disadvantage of IFS, PyFS, and qROFS is that they can only express a supportive or unsupportive attitude, and are no longer applicable when people refuse to make an evaluation. Thus, Cuong [17] developed a picture fuzzy set (PFS), where the sum of the degrees of membership, abstinence, and non-membership needs to be in the range of 0 to 1. Mahmood et al. [18] had put forward the theory of a spherical fuzzy set (SFS) and the concept of a T-spherical fuzzy set (T-SFS), which facilitates the expression of people's preferences more freely and has a broader application prospect. The above fuzzy sets are useful in evaluating uncertain and incomplete information. However, they have greater limitations due to periodic information or two-dimensional phenomena. Ramot et al. [19] gave the concept of a complex fuzzy set (CFS), extending the degree of membership from between 0 and

1 to the unit circle of the complex plane. CFS is defined as $u = re^{i2\pi(\psi_r)}$, where r represents the amplitude term lying in the interval $[0, 1]$, ψ_r represents the phase term with a value between 0 and 1. As the main mark to distinguish fuzzy sets, the phase term plays a crucial role in constructing the CFS model. Subsequent scholars proposed a complex intuitionistic fuzzy set [20] (CIFS), complex Pythagorean fuzzy set [21] (CPyFS), complex q-rung orthopair fuzzy set [22] (CqROFS), complex picture fuzzy set [23] (CPFS), and complex spherical fuzzy set [24] (CSFS). Nasir et al. [25] finally introduced the complex T-spherical fuzzy set (CT-SFS), which greatly improves the research's flexibility, applicability, and effectiveness of the research, and facilitates the subsequent research in the field of fuzzy decision-making.

The above fuzzy sets have a common feature: they all use quantitative data to express the assessment results. In real life, language is a common expression, especially when facing challenges in quantifying the situation. Experts have difficulty using precise data to evaluate information and often use language to express their preferences, such as very good, better, worse, very bad, etc. [26]. In this case, Zadeh's linguistic term set [27] (LTS) theory emerged. However, accurately characterizing the evaluation information with a single linguistic variable can be challenging. Chen et al. [28] proposed a linguistic intuitionistic fuzzy set (LIFS), which gives information on the degrees of membership and non-membership through linguistic variables and expresses the evaluation information more accurately. Qiyas et al. [29] incorporated the degree of neutrality based on this theory and gave the concept of a linguistic picture fuzzy set (LPFS). Subsequently, a linguistic spherical fuzzy set (LSFS) was defined by Jin et al. [30]. Considering the defects of the proposed fuzzy set, to solve this problem, Liu et al. [31] introduced the theory of linguistic T-spherical fuzzy set (LT-SFS), which has greater research value regarding both the accuracy of linguistic variables and the scope of restrictions. However, as the complexity of the assessment problem deepens over time, the existing fuzzy sets can no longer meet the growing demand for multi-attribute assessment. Based on the existing results, this paper combines the CT-SFS and the LTS to define the new linguistic complex T-spherical fuzzy set (LCT-SFS), providing a new decision-making assessment environment. Compared with a single fuzzy set, it can contain more comprehensive uncertainty information and meet the needs of higher-level and multi-attribute assessment.

The aggregation of assessment information is a key component in the multi-attribute assessment process. Scholars highly value the information aggregation operator as an effective means for aggregating assessment information. At present, weighted averaging (WA), weighted geometric (WG), ordered weighted averaging (OWA), ordered weighted geometric (OWG), hybrid weighted averaging (HWA), and hybrid weighted geometric (HWG) operators have been more widely used in the research of scholars. However, the above studies are all based on the assumption of independence between attributes, and correlations between different attributes can be observed in practical applications. To solve the problem of correlation existing between attributes, Bonferroni [32] proposed the Bonferroni mean (BM) operator. On this basis, Liu and Pei [33] proposed a Heronian mean (HM) operator. Through various comparative analyses, Yu and Wu [34] proved that the HM operator is better than the BM operator in information aggregation. The above aggregation operators are based on the algebraic operations of T-module and S-module operations, which lack flexibility and robustness. Dombi [35] proposed Dombi t-conorm and t-norm (DTT), which has superior characteristics in terms of information aggregation. Scholars have successively proposed picture fuzzy Dombi HM operators [36], interval-valued intuitionistic fuzzy Dombi HM operators [37], cubic fuzzy HM Dombi aggregation operators [38], and linguistic picture fuzzy Dombi HM operators [39]. The above classes of operators combine the advantages of the Dombi operations and the HM operator. They can flexibly deal with the problem of the existence of a correlation between attributes. To reduce the impact of singularities in the assessment information on the assessment results, Liu et al. [40] proposed 2-tuple linguistic neutrosophic Dombi power HM operators, while Zhang et al. [41] put forward spherical fuzzy Dombi power HM operators. The above two

classes of operators unite the combined strengths of Dombi, PA, and HM operators. To address the need for partitioned aggregation of different dimensional attributes, Zhong et al. [42] introduced the idea of partitioned and proposed q-rung orthopair fuzzy Dombi power partitioned Heronian mean (PHM) operators. According to the literature review, few studies have explored the expansion of Dombi operations combined with the PHM operators. Therefore, carrying out deeper expansion research and continuously improving the uncertainty information aggregation technology are necessary. This paper attempts to extend the Dombi power PHM operator to the newly proposed LCT-SF setting. In addition, an advanced operator is a kind of advanced aggregation technology that can solve the aggregation result's irrationality and its inability to differentiate the assessment object that occurs in the aggregation process of the basic operator. Therefore, we propose the ALCT-SFDWPPHM operator to realize the effective aggregation of multi-structural, multi-objective, and multi-dimensional uncertainty information and arrive at a more reasonable aggregation result to make a basic technical support for multi-attribute assessment work.

Information aggregation operators are commonly employed to consolidate experts' assessment information, and the optimal decision made by experts is determined through multi-attribute decision-making methods. The weighted aggregated sum product assessment (WASPAS) [43] is based on the weighted sum model (WSM) and the weighted product model (WPM) to achieve adjustable levels of decision-making accuracy. Compared with the single method, the WASPAS method can improve aggregation accuracy and make decision-making more scientific and rational. The WASPAS method has been applied to multi-attribute decision-making research because of its advantage of being able to evaluate the target object more accurately. At present, the WASPAS method has been applied in a variety of fuzzy decision-making environments, such as intuitionistic [44], Pythagorean [45], q-rung orthopair [46], picture [47], and two-tuple linguistic Fermatean fuzzy environment [48], which has a wide range of application prospects. However, no relevant research extends to the linguistic environment. This paper attempts to extend the WASPAS method to the newly proposed LCT-SF environment to solve the problem of the quality assessment of emergency information, which is a brand-new expansion of the WASPAS method in terms of the research context and the scope of the target audience.

This paper defines LCT-SFS and its underlying operations and information measures to provide theoretical support for subsequent research. We propose the ALCT-SFDWPPHM and ALCT-SFDWPPGHM operators by taking the LCT-SFS as the object of analysis, the Dombi operations as the underlying rule, and the PA, the PHM, and the advanced operators as technical elements. Further, a new multi-attribute assessment method that combines the ALCT-SFDWPPHM operator and the WASPAS method to solve the problem of the quality assessment of emergency information is proposed. The main contributions and innovations of this paper are as follows:

1. We combine the advantages of CT-SFS and LTS for characterizing uncertain information, and propose LCT-SFS, which is a novel fuzzy set that can characterize more comprehensive uncertain information, is more suitable for dealing with fuzzy multi-attribute assessment problems, and can provide a brand-new fuzzy environment for the future research of multi-attribute assessment methods.
2. We propose the ALCT-SFDWPPHM and ALCT-SFDWPPGHM operators, which can deal with the problem of aggregation of attributes of different dimensions and the correlation problem that exists between attributes of the same dimension but also eliminate the influence of singularities on the assessment results. At the same time, we avoid the situation where the aggregation results are consistent or indistinguishable. It provides greater flexibility and superiority in the aggregation process, which enhances and optimizes the uncertain information aggregation method.
3. We combine the ALCT-SFDWPPHM operator and the WASPAS method to construct a new multi-attribute assessment method, which has the advantages of being able to improve the reliability and validity of the multi-attribute assessment results and can be widely used in the practice of fuzzy multi-attribute assessment of multi-

structures, multi-dimensions, and multi-objectives, and can represent the continuous improvement of the existing multi-attribute assessment methods.

4. We constructed a hierarchical model of emergency information quality assessment indices from the user's cognition and emotional experience perspective and applied the proposed multi-attribute assessment methodology to the quality assessment of emergency information, which can provide a reliable basis for further improving the quality of emergency information and thus better assist emergency management.

The structure of this paper is as follows. Section 2 introduces some basic concepts, such as CT-SFS, LTS, Dombi, HM, PHM, and PA operators. In Section 3, we propose the ALCT-SFDWPPHM and the ALCT-SFDWPPGHM operators. At the same time, the properties of these operators are given and proved. Section 4 constructs the index system of the quality assessment of emergency information. In Section 5, we propose a new multi-attribute assessment method, which utilizes the ALCT-SFDPPWHM operator and the WASPAS method to the quality assessment of emergency information. In Section 6, the proposed multi-attribute assessment method is applied to a specific real-life example. Moreover, sensitivity analysis, comparative analysis, and discussions are conducted to demonstrate the effectiveness and superiority of the proposed method. Section 7 expounds on the conclusions.

2. Preliminaries

2.1. CT-SFS

Definition 1 [18]. Let X be a non-empty set, then the CT-SFS is defined as:

$$F = \left\{ \langle x, \tilde{\gamma}_F(x), \tilde{\delta}_F(x), \tilde{\tau}_F(x) \rangle : x \in X \right\} \quad (1)$$

where $\tilde{\gamma}_F(x) = \gamma_F(x)e^{i2\pi\sigma_{\gamma_F(x)}}$, $\tilde{\delta}_F(x) = \delta_F(x)e^{i2\pi\sigma_{\delta_F(x)}}$, $\tilde{\tau}_F(x) = \tau_F(x)e^{i2\pi\sigma_{\tau_F(x)}}$ $\in [0, 1]$.

For any x , the conditions are satisfied simultaneously:

$$\begin{aligned} 0 &\leq \gamma_F^q(x) + \delta_F^q(x) + \tau_F^q(x) \leq 1 \\ 0 &\leq \sigma^q_{\gamma_F(x)} + \sigma^q_{\delta_F(x)} + \sigma^q_{\tau_F(x)} \leq 1 \end{aligned} \quad (2)$$

where $\tilde{\gamma}_F(x)$ denotes membership degree, $\tilde{\delta}_F(x)$ denotes abstinence degree, and $\tilde{\tau}_F(x)$ denotes non-membership degree.

For complex T-spherical fuzzy number (CTSFN), when $x \in X$, the degree of hesitant fuzzy can be expressed as:

$$\zeta_F(x) = \sqrt[q]{1 - \gamma_F^q(x) - \delta_F^q(x) - \tau_F^q(x)} e^{i2\pi \sqrt[q]{1 - \sigma^q_{\gamma_F(x)} - \sigma^q_{\delta_F(x)} - \sigma^q_{\tau_F(x)}}} \quad (3)$$

For convenience, $(\gamma_F(x)e^{i2\pi\sigma_{\gamma_F(x)}}, \delta_F(x)e^{i2\pi\sigma_{\delta_F(x)}}, \tau_F(x)e^{i2\pi\sigma_{\tau_F(x)}})$ is called the CTSFN, denoted as $\mathcal{F} = (\gamma, \delta, \tau)$.

2.2. LTS

Definition 2 [27]. Let $S = \{S_t | t = 0, 1, \dots, l\}$ be a LTS with the following characteristics:

1. If $m > n$, then $S_m > S_n$;
2. If $m + n = l$, then negation $(S_m) = S_n$;
3. If $S_m \geq S_n$, then $\max(S_m, S_n) = S_m$;
4. If $S_m \leq S_n$, then $\min(S_m, S_n) = S_m$.

2.3. Dombi t-Norm and t-Conorm

Definition 3 [35]. Let $\lambda > 0, h, l \in [0, 1]$. DTT are defined as follows:

$$B_{D,\lambda}(h, l) = \frac{1}{1 + \left(\left(\frac{1-h}{h} \right)^\lambda + \left(\frac{1-l}{l} \right)^\lambda \right)^{\frac{1}{\lambda}}} \quad (4)$$

$$B_{D,\lambda}^c(h, l) = 1 - \frac{1}{1 + \left(\left(\frac{h}{1-h} \right)^\lambda + \left(\frac{l}{1-l} \right)^\lambda \right)^{\frac{1}{\lambda}}} \quad (5)$$

2.4. HM Operator

Definition 4 [33]. Let $x_i (i = 1, 2, \dots, n)$ be a series of crisp numbers, $a, b \geq 0$, then the HM operator is defined as follows:

$$HM^{a,b}(x_1, x_2, \dots, x_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=1}^n x_i^a x_j^b \right)^{\frac{1}{a+b}} \quad (6)$$

2.5. PHM Operator

Definition 5 [49]. Let $X_i (i = 1, 2, \dots, n)$ be a series of crisp numbers, which are partitioned into c partitions, O_1, O_2, \dots, O_c , respectively, where $O_f = \{K_{f1}, K_{f2}, \dots, K_{f|O_f|}\} (f = 1, 2, \dots, c)$ and $\sum_{f=1}^c |O_f| = n$. For an arbitrary $a, b \geq 0$, the PHM operator is defined as follows:

$$PHM^{a,b}(X_1, X_2, \dots, X_n) = \frac{1}{c} \left(\sum_{f=1}^c \left(\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} X_{fi}^a X_{fj}^b \right)^{\frac{1}{a+b}} \right) \quad (7)$$

2.6. PA Operator

Definition 6 [50]. Let $x_i (i = 1, 2, \dots, m)$ be non-negative real numbers, then the PA operator is defined as follows:

$$PA(x_1, x_2, \dots, x_m) = \frac{\sum_{i=1}^m ((1 + T(x_i))x_i)}{\sum_{i=1}^m (1 + T(x_i))} \quad (8)$$

where

$$T(x_i) = \sum_{j=1, j \neq i}^m \text{Sup}(x_i, x_j) \quad (9)$$

$\text{Sup}(x_i, x_j)$ represents the support of x_j for x_i , which satisfies the following conditions:

1. $\text{Sup}(x_i, x_j) \in [0, 1]$;
2. $\text{Sup}(x_i, x_j) = \text{Sup}(x_j, x_i)$;
3. If $|x_i - x_j| \leq |x_h - x_l|$, then $\text{Sup}(x_i, x_j) \geq \text{Sup}(x_h, x_l)$.

If $V_i = 1 + T(x_i)$, $w_i = \frac{V_i}{\sum_{i=1}^m V_i}$, then

$$PA(x_1, x_2, \dots, x_m) = \sum_{i=1}^m w_i x_i \quad (10)$$

3. Linguistic Complex T-Spherical Fuzzy Aggregation Operators

3.1. LCT-SFS

Definition 7. Let K be a non-empty set, then the LCT-SFS on K is defined as:

$$K = \left\{ \langle k, \tilde{S}_\gamma(k), \tilde{S}_\delta(k), \tilde{S}_\tau(k) \rangle : k \in K \right\} \quad (11)$$

where $\tilde{S}_\gamma(k) = S_{\gamma_F(k)} e^{i2\pi S_{\sigma_{\gamma_F(k)}}}$, $\tilde{S}_\delta(k) = S_{\delta_F(k)} e^{i2\pi S_{\sigma_{\delta_F(k)}}}$, $\tilde{S}_\tau(k) = S_{\tau_F(k)} e^{i2\pi S_{\sigma_{\tau_F(k)}}} \in [0, t]$.

For any k with the conditions:

$$\begin{aligned} 0 &\leq \gamma^q(k) + \delta^q(k) + \tau^q(k) \leq t^q, q \geq 1 \\ 0 &\leq \sigma^q_\gamma(k) + \sigma^q_\delta(k) + \sigma^q_\tau(k) \leq t^q, q \geq 1 \end{aligned} \quad (12)$$

where $\tilde{S}_\gamma(k)$ denotes linguistic membership degree, $\tilde{S}_\delta(k)$ denotes linguistic abstinence degree, and $\tilde{S}_\tau(k)$ denotes linguistic non-membership degree.

For linguistic complex T-spherical fuzzy number (LCTSFN), when $k \in K$, the refusal degree can be expressed as:

$$S_{\zeta(k)} = \sqrt[q]{t^q - \gamma^q(k) - \delta^q(k) - \tau^q(k)} e^{i2\pi \sqrt[q]{t^q - \sigma^q_\gamma(k) - \sigma^q_\delta(k) - \sigma^q_\tau(k)}} \quad (13)$$

For convenience, $(S_{\gamma(k)} e^{i2\pi S_{\sigma_{\gamma(k)}}}, S_{\delta(k)} e^{i2\pi S_{\sigma_{\delta(k)}}}, S_{\tau(k)} e^{i2\pi S_{\sigma_{\tau(k)}}})$ is called the LCTSFN, denoted as $K = (S_\gamma e^{i2\pi S_{\sigma_\gamma}}, S_\delta e^{i2\pi S_{\sigma_\delta}}, S_\tau e^{i2\pi S_{\sigma_\tau}})$.

Two LCTSFNs can be compared based on the score function and the accuracy function. The score function and accuracy function for LCTSFNs are defined as follows:

Definition 8. Let $K = (S_\gamma e^{i2\pi S_{\sigma_\gamma}}, S_\delta e^{i2\pi S_{\sigma_\delta}}, S_\tau e^{i2\pi S_{\sigma_\tau}})$ be a LCTSFN, then its score function is:

$$S(K) = S_{\left(\frac{2t^q + \gamma^q - \tau^q + \sigma_\gamma^q - \sigma_\tau^q}{4}\right)^{\frac{1}{q}}} \quad (14)$$

Definition 9. Let $K = (S_\gamma e^{i2\pi S_{\sigma_\gamma}}, S_\delta e^{i2\pi S_{\sigma_\delta}}, S_\tau e^{i2\pi S_{\sigma_\tau}})$ be a LCTSFN, then its accuracy function is:

$$A(K) = S_{(\gamma^q + \delta^q + \tau^q + \sigma_\gamma^q + \sigma_\delta^q + \sigma_\tau^q)^{\frac{1}{q}}} \quad (15)$$

Definition 10. Let $K_1 = (S_{\gamma_1}, S_{\delta_1}, S_{\tau_1})$, $K_2 = (S_{\gamma_2}, S_{\delta_2}, S_{\tau_2})$ be two arbitrary LCTSFNs. $S(K_1), S(K_2)$ are score functions of K_1, K_2 , respectively; $A(K_1), A(K_2)$ are accuracy functions of K_1, K_2 , respectively. Then

1. If $S(K_1) > S(K_2)$, then $K_1 > K_2$;
2. If $S(K_1) < S(K_2)$, then $K_1 < K_2$;
3. If $S(K_1) = S(K_2)$, then:
 - (1) If $A(K_1) > A(K_2)$, then $K_1 > K_2$;
 - (2) If $A(K_1) < A(K_2)$, then $K_1 < K_2$;
 - (3) If $A(K_1) = A(K_2)$, then $K_1 = K_2$.

Definition 11. Let $K_1 = (S_{\gamma_1}, S_{\delta_1}, S_{\tau_1})$, $K_2 = (S_{\gamma_2}, S_{\delta_2}, S_{\tau_2})$ be two arbitrary LCTSFNs, then the Hamming distance between K_1 and K_2 is:

$$d(K_1, K_2) = \frac{1}{4k^q} (|\gamma_1^q - \gamma_2^q| + |\delta_1^q - \delta_2^q| + |\tau_1^q - \tau_2^q| + |\pi_1^q - \pi_2^q| + |\sigma_{\gamma_1}^q - \sigma_{\gamma_2}^q| + |\sigma_{\delta_1}^q - \sigma_{\delta_2}^q| + |\sigma_{\tau_1}^q - \sigma_{\tau_2}^q| + |\sigma_{\pi_1}^q - \sigma_{\pi_2}^q|) \quad (16)$$

3.2. Dombi Operations for Linguistic Complex T-Spherical Fuzzy Set

Definition 12. Let $K = (S_\gamma, S_\delta, S_\tau)$, $K_1 = (S_{\gamma_1}, S_{\delta_1}, S_{\tau_1})$, $K_2 = (S_{\gamma_2}, S_{\delta_2}, S_{\tau_2})$ be three arbitrary LCTSFNs, $\lambda > 0$, $S_\gamma, S_\delta, S_\tau \in [0, t]$. The operational rules of the linguistic complex T-spherical fuzzy set based on Dombi operations are defined as follows:

Let $\theta(\cdot) = \left(\frac{t^q}{t^q - \cdot}\right)^\lambda$, $\eta(\cdot) = \left(\frac{t^q - \cdot}{t^q}\right)^\lambda$, then

$$K_1 \oplus K_2 = \left(S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\theta(\gamma_1) + \theta(\gamma_2))^\frac{1}{\lambda}}}}{1 + (\theta(\gamma_1) + \theta(\gamma_2))^\frac{1}{\lambda}}}, S \sqrt[q]{\frac{t^q}{1 + (\eta(\delta_1) + \eta(\delta_2))^\frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\theta(\tau_1) + \theta(\tau_2))^\frac{1}{\lambda}}}}{1 + (\theta(\tau_1) + \theta(\tau_2))^\frac{1}{\lambda}}}} \right) \quad (17)$$

$$K_1 \otimes K_2 = \left(S \sqrt[q]{\frac{t^q}{1 + (\eta(\gamma_1) + \eta(\gamma_2))^\frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\theta(\delta_1) + \theta(\delta_2))^\frac{1}{\lambda}}}}{1 + (\theta(\delta_1) + \theta(\delta_2))^\frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\theta(\tau_1) + \theta(\tau_2))^\frac{1}{\lambda}}}}{1 + (\theta(\tau_1) + \theta(\tau_2))^\frac{1}{\lambda}}}} \right) \quad (18)$$

$$\omega K = \left(S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\omega\theta(\gamma))^\frac{1}{\lambda}}}}{1 + (\omega\theta(\gamma))^\frac{1}{\lambda}}}, S \sqrt[q]{\frac{t^q}{1 + (\omega\eta(\delta))^\frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\omega\theta(\tau))^\frac{1}{\lambda}}}}{1 + (\omega\theta(\tau))^\frac{1}{\lambda}}} \right) \quad (19)$$

$$K^\phi = \left(S \sqrt[q]{\frac{t^q}{1 + (\phi\eta(\gamma))^\frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\phi\theta(\delta))^\frac{1}{\lambda}}}}{1 + (\phi\theta(\delta))^\frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q - \frac{t^q}{1 + (\phi\theta(\tau))^\frac{1}{\lambda}}}}{1 + (\phi\theta(\tau))^\frac{1}{\lambda}}} \right) \quad (20)$$

Theorem 1. Let $K = (S_\gamma, S_\delta, S_\tau)$, $K_1 = (S_{\gamma_1}, S_{\delta_1}, S_{\tau_1})$, $K_2 = (S_{\gamma_2}, S_{\delta_2}, S_{\tau_2})$ be three arbitrary LCTSFNs. $\psi, \psi_1, \psi_2 > 0$, then

$$K_1 \oplus K_2 = K_2 \oplus K_1 \quad (21)$$

$$K_1 \otimes K_2 = K_2 \otimes K_1 \quad (22)$$

$$\psi(K_1 \oplus K_2) = \psi K_1 \oplus \psi K_2 \quad (23)$$

$$(\psi_1 + \psi_2)K = \psi_1 K \oplus \psi_2 K \quad (24)$$

$$(K_1 \otimes K_2)^\psi = K_1^\psi \otimes K_2^\psi \quad (25)$$

$$K^{\psi_1} \otimes K^{\psi_2} = K^{(\psi_1 + \psi_2)} \quad (26)$$

Proof.

Let $\theta(\mathcal{K}) = \left(\frac{t^q}{t^q - \mathcal{K}}\right)^\lambda$, $\eta(\mathcal{K}) = \left(\frac{t^q - \mathcal{K}}{t^q}\right)^\lambda$, then

$$\begin{aligned}
K_1 \oplus K_2 &= \left(S \sqrt[q]{t^q - \frac{t^q}{1 + (\theta(\gamma_1) + \theta(\gamma_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\theta(\sigma_{\gamma_1}) + \theta(\sigma_{\gamma_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\eta(\delta_1) + \eta(\delta_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\eta(\sigma_{\delta_1}) + \eta(\sigma_{\delta_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\eta(\tau_1) + \eta(\tau_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\eta(\sigma_{\tau_1}) + \eta(\sigma_{\tau_2})) \frac{1}{\lambda}}}} \right) \\
&= \left(S \sqrt[q]{\frac{t^q}{1 + (\theta(\gamma_2) + \theta(\gamma_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\theta(\sigma_{\gamma_2}) + \theta(\sigma_{\gamma_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\eta(\delta_2) + \eta(\delta_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\eta(\sigma_{\delta_2}) + \eta(\sigma_{\delta_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\eta(\tau_2) + \eta(\tau_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\eta(\sigma_{\tau_2}) + \eta(\sigma_{\tau_1})) \frac{1}{\lambda}}}} \right) \\
&= K_2 \oplus K_1 \\
\\
K_1 \otimes K_2 &= \left(S \sqrt[q]{\frac{t^q}{1 + (\eta(\gamma_1) + \eta(\gamma_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\eta(\sigma_{\gamma_1}) + \eta(\sigma_{\gamma_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\theta(\delta_1) + \theta(\delta_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\theta(\sigma_{\delta_1}) + \theta(\sigma_{\delta_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\theta(\tau_1) + \theta(\tau_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\theta(\sigma_{\tau_1}) + \theta(\sigma_{\tau_2})) \frac{1}{\lambda}}}} \right) \\
&= \left(S \sqrt[q]{\frac{t^q}{1 + (\eta(\gamma_2) + \eta(\gamma_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\eta(\sigma_{\gamma_2}) + \eta(\sigma_{\gamma_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\theta(\delta_2) + \theta(\delta_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\theta(\sigma_{\delta_2}) + \theta(\sigma_{\delta_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\theta(\tau_2) + \theta(\tau_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\theta(\sigma_{\tau_2}) + \theta(\sigma_{\tau_1})) \frac{1}{\lambda}}}} \right) \\
&= K_2 \otimes K_1 \\
\\
\psi(K_1 \oplus K_2) &= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\gamma_1) + \psi\theta(\gamma_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\gamma_1}) + \psi\theta(\sigma_{\gamma_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\delta_1) + \psi\eta(\delta_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\delta_1}) + \psi\eta(\sigma_{\delta_2})) \frac{1}{\lambda}}}}, \right. \\
&\quad \left. S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\tau_1) + \psi\eta(\tau_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\tau_1}) + \psi\eta(\sigma_{\tau_2})) \frac{1}{\lambda}}}} \right) \\
&= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\gamma_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\gamma_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\delta_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\delta_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\tau_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\tau_1})) \frac{1}{\lambda}}}} \right) \\
&\quad \oplus \left(S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\gamma_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\gamma_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\delta_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\delta_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\tau_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\tau_2})) \frac{1}{\lambda}}}} \right) = \psi K_1 \oplus \psi K_2 \\
\\
(\psi_1 + \psi_2)K &= \left(S \sqrt[q]{\frac{t^q}{1 + ((\psi_1 + \psi_2)\theta(\gamma)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + ((\psi_1 + \psi_2)\theta(\sigma_\gamma)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + ((\psi_1 + \psi_2)\eta(\delta)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + ((\psi_1 + \psi_2)\eta(\sigma_\delta)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + ((\psi_1 + \psi_2)\eta(\tau)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + ((\psi_1 + \psi_2)\eta(\sigma_\tau)) \frac{1}{\lambda}}}} \right) \\
&= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\gamma)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\sigma_\gamma)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\delta)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\sigma_\delta)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\tau)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\sigma_\tau)) \frac{1}{\lambda}}}} \right) \\
&\quad \oplus \left(S \sqrt[q]{\frac{t^q}{1 + (\psi_2\theta(\gamma)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi_2\theta(\sigma_\gamma)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_2\eta(\delta)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi_2\eta(\sigma_\delta)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_2\eta(\tau)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{q} \sqrt[q]{\frac{t^q}{1 + (\psi_2\eta(\sigma_\tau)) \frac{1}{\lambda}}}} \right) = \psi_1 K \oplus \psi_2 K
\end{aligned}$$

$$\begin{aligned}
(K_1 \otimes K_2)^\psi &= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\gamma_1) + \psi\eta(\gamma_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\gamma_1}) + \psi\eta(\sigma_{\gamma_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\delta_1) + \psi\theta(\delta_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\delta_1}) + \psi\theta(\sigma_{\delta_2})) \frac{1}{\lambda}}}}, \right. \\
&\quad \left. S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\tau_1) + \psi\theta(\tau_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\tau_1}) + \psi\theta(\sigma_{\tau_2})) \frac{1}{\lambda}}}} \right) \\
&= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\gamma_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\gamma_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\delta_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\delta_1})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\tau_1)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\tau_1})) \frac{1}{\lambda}}}}, \right. \\
&\quad \left. \otimes \left(S \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\gamma_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\eta(\sigma_{\gamma_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\delta_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\delta_2})) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\tau_2)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi\theta(\sigma_{\tau_2})) \frac{1}{\lambda}}}} \right) \right) = K_1^\psi \otimes K_2^\psi \\
K^{\psi_1} \otimes K^{\psi_2} &= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\gamma)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\sigma_\gamma)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\delta)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\sigma_\delta)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\tau)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\sigma_\tau)) \frac{1}{\lambda}}}}, \right. \\
&\quad \left. \otimes \left(S \sqrt[q]{\frac{t^q}{1 + (\psi_2\eta(\gamma)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_2\eta(\sigma_\gamma)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_2\theta(\delta)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_2\theta(\sigma_\delta)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_2\theta(\tau)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_2\theta(\sigma_\tau)) \frac{1}{\lambda}}}} \right) \right) \\
&= \left(S \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\gamma) + \psi_2\eta(\gamma)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_1\eta(\sigma_\gamma) + \psi_2\eta(\sigma_\gamma)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\delta) + \psi_2\theta(\delta)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\sigma_\delta) + \psi_2\theta(\sigma_\delta)) \frac{1}{\lambda}}}}, S \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\tau) + \psi_2\theta(\tau)) \frac{1}{\lambda}}} e^{\frac{i2\pi S}{i2\pi S} \sqrt[q]{\frac{t^q}{1 + (\psi_1\theta(\sigma_\tau) + \psi_2\theta(\sigma_\tau)) \frac{1}{\lambda}}}} \right) \\
&= K^{\psi_1 + \psi_2}
\end{aligned}$$

□

3.3. Advanced Linguistic Complex T-Spherical Fuzzy Dombi-Weighted Power-Partitioned Heronian Mean Operator

Definition 13. Let $a, b \geq 0, \lambda > 0, K_i = (S_{\gamma_i}, S_{\delta_i}, S_{\tau_i}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs that are partitioned into c partitions, O_1, O_2, \dots, O_c , respectively, where $O_f = \{K_{f1}, K_{f2}, \dots, K_{f|O_f|}\}$ ($f = 1, 2, \dots, c$) and $\sum_{f=1}^c |O_f| = n$. If w_i denotes the weight of K_i , where $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$. For any $a, b \geq 0$, and a, b are not 0 simultaneously. The ALCT-SFDWPPHM operator is defined as follows:

$$ALCT-SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) = \frac{1}{c} \left(\bigoplus_{f=1}^c \left(\frac{2}{|O_f|(|O_f|+1)} \bigoplus_{i=1}^{|O_f|} \bigoplus_{j=i}^{|O_f|} \left(\frac{nw_{fi}(1+T(K_{fi}))}{\sum_{g=1}^n w_g(1+T(K_g))} K_i \right)^a \otimes \left(\frac{nw_{fj}(1+T(K_{fj}))}{\sum_{g=1}^n w_g(1+T(K_g))} K_j \right)^b \right) \right)^{\frac{1}{a+b}} \quad (27)$$

where $T(K_i) = \sum_{j=1, j \neq i}^n \text{Sup}(K_i, K_j)$, $\text{Sup}(K_i, K_j) = 1 - d(K_i, K_j)$, $\text{Sup}(K_i, K_j)$ indicates the support of K_j to K_i , which satisfies the following conditions:

1. $\text{Sup}(x_i, x_j) \in [0, 1]$;
2. $\text{Sup}(x_i, x_j) = \text{Sup}(x_j, x_i)$;
3. If $d(K_i, K_j) \leq d(K_m, K_q)$, then $\text{Sup}(x_i, x_j) \geq \text{Sup}(K_m, K_q)$.

Let

$$w'_i = \frac{1 + T(K_{fi})}{\sum_{i=1}^n (1 + T(K_g))} \quad (28)$$

where, $w'_i \in [0, 1]$, and $\sum_{i=1}^n w'_i = 1$, then

$$ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) = \frac{1}{c} \left(\bigoplus_{f=1}^c \left(\frac{2}{|O_f|(|O_f|+1)} \bigoplus_{i=1}^{|O_f|} \bigoplus_{j=i}^{|O_f|} \left(\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} \right)^a \otimes \left(\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} \right)^b \right) \right)^{\frac{1}{a+b}} \quad (29)$$

Theorem 2. Let $K_i = (S_{\gamma_i}, S_{\delta_i}, S_{\tau_i}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs, divided into c partitions as $O_1, O_2, \dots, O_c, O_f = \{K_{f1}, K_{f2}, \dots, K_{f|O_f|}\} (f = 1, 2, \dots, c)$ and $\sum_{f=1}^c |O_f| = n$. If w_i denotes the weight of K_i , where $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$. For any $a, b \geq 0$, and a, b are not 0 simultaneously, $\lambda > 0$. Then, applying the ALCT-SFDWPPHM operator to aggregate, the result is still a LCTSFN, which can be expressed as:

$$ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) = \left(S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\{\gamma_{fi}\} + b/\{\gamma_{fj}\}}))^\lambda)} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\{\gamma_{fi}\} + b/\{\gamma_{fj}\}}))^\lambda)}}, S \sqrt[\frac{1}{\rho}]{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\{\delta_{fi}\} + b/\{\delta_{fj}\}}))^\lambda)} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\{\delta_{fi}\} + b/\{\delta_{fj}\}}))^\lambda)}}, S \sqrt[\frac{1}{\rho}]{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\{\tau_{fi}\} + b/\{\tau_{fj}\}}))^\lambda)} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\{\tau_{fi}\} + b/\{\tau_{fj}\}}))^\lambda)}} \right) \quad (30)$$

$$\text{where } \varrho(h_{fi}) = \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fi})^q}{t^q - (\rho h_{fi})^q} \right)^\lambda, \varrho(h_{fj}) = \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fj})^q}{t^q - (\rho h_{fj})^q} \right)^\lambda, \vartheta(h_{fi}) = \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fi})^q}{(\rho h_{fi})^q} \right)^\lambda, \vartheta(h_{fj}) = \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fj})^q}{(\rho h_{fj})^q} \right)^\lambda.$$

Proof.

$$\begin{aligned} \text{Let } \varrho(h_{fi}) &= \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fi})^q}{t^q - (\rho h_{fi})^q} \right)^\lambda, \varrho(h_{fj}) = \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fj})^q}{t^q - (\rho h_{fj})^q} \right)^\lambda, \vartheta(h_{fi}) = \\ &= \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fi})^q}{(\rho h_{fi})^q} \right)^\lambda, \vartheta(h_{fj}) = \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fj})^q}{(\rho h_{fj})^q} \right)^\lambda \\ \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} &= \left(S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \{\gamma_{fi}\})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \{\gamma_{fi}\})}}, S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\delta_{fi}\})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\delta_{fi}\})}}, S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\tau_{fi}\})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\tau_{fi}\})}} \right) \\ \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} &= \left(S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \{\gamma_{fj}\})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \{\gamma_{fj}\})}}, S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\delta_{fj}\})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\delta_{fj}\})}}, S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\tau_{fj}\})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q / (1 + \{\tau_{fj}\})}} \right) \\ \left(\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} \right)^a &= \left(S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{a}{\{\gamma_{fi}\}})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{a}{\{\gamma_{fi}\}})}}, S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{a}{\{\delta_{fi}\}})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{a}{\{\delta_{fi}\}})}}, S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{a}{\{\tau_{fi}\}})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{a}{\{\tau_{fi}\}})}} \right) \\ \left(\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} \right)^b &= \left(S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{b}{\{\gamma_{fj}\}})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{b}{\{\gamma_{fj}\}})}}, S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{b}{\{\delta_{fj}\}})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{b}{\{\delta_{fj}\}})}}, S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{b}{\{\tau_{fj}\}})} e^{i2\pi S \sqrt[\frac{1}{\rho}]{t^q - t^q / (1 + \frac{b}{\{\tau_{fj}\}})}} \right) \end{aligned}$$

$$\begin{aligned}
& \left(\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} \right)^a \otimes \left(\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} \right)^b \\
&= \left(S \frac{1}{\rho} \sqrt[1]{t^q / (1 + (\frac{a}{\theta(\gamma_{fi})} + \frac{b}{\theta(\gamma_{fj})})^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q / (1 + (\frac{a}{\theta(\gamma_{fi})} + \frac{b}{\theta(\gamma_{fj})})^{\frac{1}{\lambda}})}} \right)^{\frac{1}{\lambda}} \\
& \quad S \frac{1}{\rho} \sqrt[1]{t^q - t^q / (1 + (\frac{a}{\theta(\gamma_{fi})} + \frac{b}{\theta(\gamma_{fj})})^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q - t^q / (1 + (\frac{a}{\theta(\gamma_{fi})} + \frac{b}{\theta(\gamma_{fj})})^{\frac{1}{\lambda}})}} \\
&= \left(\frac{2}{|O_f|(|O_f|+1)} \oplus_{i=1}^{|O_f|} \oplus_{j=i}^{|O_f|} \left(\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} \right)^a \otimes \left(\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} \right)^b \right)^{\frac{1}{a+b}} \\
&= \left(S \frac{1}{\rho} \sqrt[1]{t^q - t^q / (1 + (\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})})^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q - t^q / (1 + (\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})})^{\frac{1}{\lambda}})}} \right)^{\frac{1}{\lambda}} \\
& \quad S \frac{1}{\rho} \sqrt[1]{t^q / (1 + (\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})})^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q / (1 + (\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})})^{\frac{1}{\lambda}})}} \\
& \quad S \frac{1}{\rho} \sqrt[1]{t^q / (1 + (\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})})^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q / (1 + (\frac{2}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})})^{\frac{1}{\lambda}})}} \\
& \quad \left(\oplus_{f=1}^c \left(\frac{2}{|O_f|(|O_f|+1)} \oplus_{i=1}^{|O_f|} \oplus_{j=i}^{|O_f|} \left(\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} \right)^a \otimes \left(\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} \right)^b \right) \right)^{\frac{1}{a+b}} \\
&= \left(S \frac{1}{\rho} \sqrt[1]{t^q / (1 + (1/\sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})}))^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q / (1 + (1/\sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})}))^{\frac{1}{\lambda}})}} \right)^{\frac{1}{\lambda}} \\
& \quad S \frac{1}{\rho} \sqrt[1]{t^q - t^q / (1 + (1/\sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})}))^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q - t^q / (1 + (1/\sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})}))^{\frac{1}{\lambda}})}} \\
& \quad S \frac{1}{\rho} \sqrt[1]{t^q - t^q / (1 + (1/\sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})}))^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q - t^q / (1 + (1/\sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})}))^{\frac{1}{\lambda}})}} \\
&= \left(\frac{1}{c} \left(\oplus_{f=1}^c \left(\frac{2}{|O_f|(|O_f|+1)} \oplus_{i=1}^{|O_f|} \oplus_{j=i}^{|O_f|} \left(\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} K_{fi} \right)^a \otimes \left(\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} K_{fj} \right)^b \right) \right) \right)^{\frac{1}{a+b}} \\
&= \left(S \frac{1}{\rho} \sqrt[1]{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})}))^{\frac{1}{\lambda}})} e^{\frac{i2\pi S}{\rho} \sqrt[1]{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})}))^{\frac{1}{\lambda}})}} \right)^{\frac{1}{\lambda}}
\end{aligned}$$

$$S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})}))^{\frac{1}{\lambda}})}]}},$$

$$S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})}))^{\frac{1}{\lambda}})}]} \Bigg)$$

Thus, the conclusion is proved. \square

Theorem 3 (Idempotency). Let $K_i = (S_{\gamma_i}, S_{\delta_i}, S_{\tau_i}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs, where $a, b \geq 0$ and a, b are not 0 simultaneously.

For all $i = 1, 2, \dots, n$, if $K_i = K = (S_{\gamma}, S_{\delta}, S_{\tau})$, then

$$ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) = K \quad (31)$$

Proof.

Let $\aleph(u) = \left(\frac{(\rho u)^q}{t^q - (\rho u)^q} \right)^\lambda$, $\mathcal{H}(u) = \left(\frac{t^q - (\rho u)^q}{(\rho u)^q} \right)^\lambda$. For all $i = 1, 2, \dots, n$, $K_i = K = (S_{\gamma}, S_{\delta}, S_{\tau})$, $\sum_{i=1}^n w_i = 1$, then $w_i = \frac{1}{n}$.

$$\begin{aligned} & ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \\ &= ALCT - SFDWPPHM^{a,b}(K, K, \dots, K) = \\ &= \left(S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\aleph(\gamma)) + b/(n*\frac{1}{n}\aleph(\gamma))}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\aleph(\gamma)) + b/(n*\frac{1}{n}\aleph(\gamma))}))^{\frac{1}{\lambda}})}]}}, \right. \\ & S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\delta)) + b/(n*\frac{1}{n}\mathcal{H}(\delta))}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\delta)) + b/(n*\frac{1}{n}\mathcal{H}(\delta))}))^{\frac{1}{\lambda}})}]}}, \\ & S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\tau)) + b/(n*\frac{1}{n}\mathcal{H}(\tau))}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\tau)) + b/(n*\frac{1}{n}\mathcal{H}(\tau))}))^{\frac{1}{\lambda}})}]} \Bigg) \\ &= \left(S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\aleph(\gamma)) + b/(n*\frac{1}{n}\aleph(\gamma))}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\aleph(\gamma)) + b/(n*\frac{1}{n}\aleph(\gamma))}))^{\frac{1}{\lambda}})}]}}, \right. \\ & S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\delta)) + b/(n*\frac{1}{n}\mathcal{H}(\delta))}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\delta)) + b/(n*\frac{1}{n}\mathcal{H}(\delta))}))^{\frac{1}{\lambda}})}]}}, \\ & S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\tau)) + b/(n*\frac{1}{n}\mathcal{H}(\tau))}))^{\frac{1}{\lambda}})}]{e^{i2\pi S \sqrt[t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/(n*\frac{1}{n}\mathcal{H}(\tau)) + b/(n*\frac{1}{n}\mathcal{H}(\tau))}))^{\frac{1}{\lambda}})}]} \Bigg) \end{aligned}$$

$$\begin{aligned}
&= \left(S_{\frac{1}{p} \sqrt{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c \frac{\mathcal{N}(\gamma)}{a+b})^{\frac{1}{\lambda}})}} e^{\frac{i2\pi S}{p} \sqrt{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c \frac{\mathcal{N}(\gamma)}{a+b})^{\frac{1}{\lambda}})}}, S_{\frac{1}{p} \sqrt{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c \frac{\mathcal{H}(\delta)}{a+b})^{\frac{1}{\lambda}})}} e^{\frac{i2\pi S}{p} \sqrt{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c \frac{\mathcal{H}(\delta)}{a+b})^{\frac{1}{\lambda}})}}, \right. \\
&\quad \left. S_{\frac{1}{p} \sqrt{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c \frac{\mathcal{H}(\tau)}{a+b})^{\frac{1}{\lambda}})}} e^{\frac{i2\pi S}{p} \sqrt{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c \frac{\mathcal{H}(\tau)}{a+b})^{\frac{1}{\lambda}})}} \right) \\
&= (S_\gamma, S_\delta, S_\tau) \\
&= K
\end{aligned}$$

□

Theorem 4 (Monotonicity). Let $K_i = (S_{\gamma_i} e^{i2\pi S_{\sigma_{\gamma_i}}}, S_{\delta_i} e^{i2\pi S_{\sigma_{\delta_i}}}, S_{\tau_i} e^{i2\pi S_{\sigma_{\tau_i}}})$, $K_i' = (S_{\gamma_i'} e^{i2\pi S_{\sigma_{\gamma_i'}}}, S_{\delta_i'} e^{i2\pi S_{\sigma_{\delta_i'}}}, S_{\tau_i'} e^{i2\pi S_{\sigma_{\tau_i'}}})$ ($i = 1, 2, \dots, n$) be two LCTSFNs, where $a, b \geq 0$ and a, b are not 0 simultaneously. For all $i = 1, 2, \dots, n$, If $S_{\gamma(f_i)} \leq S_{\gamma(f_i')}, S_{\sigma_{\gamma(f_i)}} \leq S_{\sigma_{\gamma(f_i')}}, S_{\delta(f_i)} \geq S_{\delta(f_i')}, S_{\sigma_{\delta(f_i)}} \geq S_{\sigma_{\delta(f_i')}}, S_{\tau(f_i)} \geq S_{\tau(f_i')}, S_{\sigma_{\tau(f_i)}} \geq S_{\sigma_{\tau(f_i)'}}$, then

$$ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \leq ALCT - SFDWPPHM^{a,b}(K_1', K_2', \dots, K_n') \quad (32)$$

Proof.

Let

$$\begin{aligned}
ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) &= K_i = (S_{\gamma_i} e^{i2\pi S_{\sigma_{\gamma_i}}}, S_{\delta_i} e^{i2\pi S_{\sigma_{\delta_i}}}, S_{\tau_i} e^{i2\pi S_{\sigma_{\tau_i}}}) \\
ALCT - SFDWPPHM^{a,b}(K_1', K_2', \dots, K_n') &= K_i' = (S_{\gamma_i'} e^{i2\pi S_{\sigma_{\gamma_i'}}}, S_{\delta_i'} e^{i2\pi S_{\sigma_{\delta_i'}}}, S_{\tau_i'} e^{i2\pi S_{\sigma_{\tau_i'}}}) \\
\mathcal{G}(h_{fi}) &= \frac{nw_{fi}' w_{fi}}{\sum_{g=1}^n w_g w_g'} \left(\frac{(\rho h_{fi})^q}{t^q - (\rho h_{fi})^q} \right)^\lambda, \mathcal{G}(h_{fj}) = \frac{nw_{fj}' w_{fj}}{\sum_{g=1}^n w_g w_g'} \left(\frac{(\rho h_{fj})^q}{t^q - (\rho h_{fj})^q} \right)^\lambda, \mathcal{I}(h_{fi'}) = \frac{nw_{fi}' w_{fi}}{\sum_{g=1}^n w_g w_g'} \left(\frac{t^q - (\rho h_{fi'})^q}{(\rho h_{fi'})^q} \right)^\lambda, \mathcal{I}(h_{fj'}) = \frac{nw_{fj}' w_{fj}}{\sum_{g=1}^n w_g w_g'} \left(\frac{t^q - (\rho h_{fj'})^q}{(\rho h_{fj'})^q} \right)^\lambda.
\end{aligned}$$

Since all $i = 1, 2, \dots, n; j = 1, 2, \dots, n, S_{\gamma(f_i)} \leq S_{\gamma(f_i')}, S_{\gamma(f_j)} \leq S_{\gamma(f_j')}$, then

$$\begin{aligned}
&\frac{(\rho \gamma_{fi})^q}{t^q - (\rho \gamma_{fi})^q} \leq \frac{(\rho \gamma_{fi'})^q}{t^q - (\rho \gamma_{fi'})^q}, \frac{(\rho \gamma_{fj})^q}{t^q - (\rho \gamma_{fj})^q} \leq \frac{(\rho \gamma_{fj'})^q}{t^q - (\rho \gamma_{fj'})^q} \\
&\left(\frac{(\rho \gamma_{fi})^q}{t^q - (\rho \gamma_{fi})^q} \right)^\lambda \leq \left(\frac{(\rho \gamma_{fi'})^q}{t^q - (\rho \gamma_{fi'})^q} \right)^\lambda, \left(\frac{(\rho \gamma_{fj})^q}{t^q - (\rho \gamma_{fj})^q} \right)^\lambda \leq \left(\frac{(\rho \gamma_{fj'})^q}{t^q - (\rho \gamma_{fj'})^q} \right)^\lambda \\
&a/\mathcal{G}(\gamma_{fi}) + b/\mathcal{G}(\gamma_{fj}) \geq a/\mathcal{I}(\gamma_{fi'}) + b/\mathcal{I}(\gamma_{fj'}) \\
&\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{G}(\gamma_{fi})} + \frac{b}{\mathcal{G}(\gamma_{fj})} \right) \leq \frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{I}(\gamma_{fi'})} + \frac{b}{\mathcal{I}(\gamma_{fj'})} \right) \\
&\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{G}(\gamma_{fi})} + \frac{b}{\mathcal{G}(\gamma_{fj})} \right) \right) \leq \frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{I}(\gamma_{fi'})} + \frac{b}{\mathcal{I}(\gamma_{fj'})} \right) \right) \\
&1 + \left(\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{G}(\gamma_{fi})} + \frac{b}{\mathcal{G}(\gamma_{fj})} \right) \right) \right)^\lambda \leq 1 + \left(\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{I}(\gamma_{fi'})} + \frac{b}{\mathcal{I}(\gamma_{fj'})} \right) \right) \right)^\lambda \\
&t^q - t^q / \left(1 + \left(\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{G}(\gamma_{fi})} + \frac{b}{\mathcal{G}(\gamma_{fj})} \right) \right) \right)^\lambda \right) \leq t^q - t^q / \left(1 + \left(\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{I}(\gamma_{fi'})} + \frac{b}{\mathcal{I}(\gamma_{fj'})} \right) \right) \right)^\lambda \right) \\
&\frac{1}{p} \sqrt{t^q - t^q / \left(1 + \left(\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{G}(\gamma_{fi})} + \frac{b}{\mathcal{G}(\gamma_{fj})} \right) \right) \right)^\lambda \right)} \leq \frac{1}{p} \sqrt{t^q - t^q / \left(1 + \left(\frac{1}{c} \sum_{f=1}^c \left(\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=1}^{|O_f|} 1/\left(\frac{a}{\mathcal{I}(\gamma_{fi'})} + \frac{b}{\mathcal{I}(\gamma_{fj'})} \right) \right) \right)^\lambda \right)}
\end{aligned}$$

Thus, $S_{\gamma_i} \leq S_{\gamma_i'}$. Similarly, it can be proved that $S_{\sigma_{\gamma_i}} \leq S_{\sigma_{\gamma_i'}}, S_{\delta_i} \geq S_{\delta_i'}, S_{\sigma_{\delta_i}} \geq S_{\sigma_{\delta_i'}}, S_{\tau_i} \geq S_{\tau_i'}, S_{\sigma_{\tau_i}} \geq S_{\sigma_{\tau_i'}}$.

Thus,

$$ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \leq ALCT - SFDWPPHM^{a,b}(K_1', K_2', \dots, K_n')$$

□

Theorem 5 (Boundedness). Let $K_i = (S_{\gamma_i} e^{i2\pi S_{\sigma\gamma_i}}, S_{\delta_i} e^{i2\pi S_{\sigma\delta_i}}, S_{\tau_i} e^{i2\pi S_{\sigma\tau_i}}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs, where $a, b \geq 0$ and a, b are not 0 simultaneously. $K^+ = (\max S_{\gamma_i} e^{i2\pi \max S_{\sigma\gamma_i}}, \min S_{\delta_i} e^{i2\pi \min S_{\sigma\delta_i}}, \min S_{\tau_i} e^{i2\pi \min S_{\sigma\tau_i}})$, $K^- = (\min S_{\gamma_i} e^{i2\pi \min S_{\sigma\gamma_i}}, \max S_{\delta_i} e^{i2\pi \max S_{\sigma\delta_i}}, \max S_{\tau_i} e^{i2\pi \max S_{\sigma\tau_i}})$, then

$$K^- \leq ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \leq K^+ \quad (33)$$

Proof.

From Theorem 3, we know that

$$ALCT - SFDWPPHM^{a,b}(K_1^-, K_2^-, \dots, K_n^-) = K^-, ALCT - SFDWPPHM^{a,b}(K_1^+, K_2^+, \dots, K_n^+) = K^+$$

From Theorem 4, we know that

$$ALCT - SFDWPPHM^{a,b}(K_1^-, K_2^-, \dots, K_n^-) \leq ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \leq ALCT - SFDWPPHM^{a,b}(K_1^+, K_2^+, \dots, K_n^+)$$

Thus,

$$K^- \leq ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \leq K^+$$

□

3.4. Advanced Linguistic Complex T-Spherical Fuzzy Dombi-Weighted Power-Partitioned Geometric Heronian Mean Operator

Definition 14. Let $a, b \geq 0, \lambda > 0, K_i = (S_{\gamma_i}, S_{\delta_i}, S_{\tau_i}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs that are partitioned into c partitions, O_1, O_2, \dots, O_c , respectively, where $O_f = \{K_{f1}, K_{f2}, \dots, K_{f|O_f|}\} (f = 1, 2, \dots, c)$ and $\sum_{f=1}^c |O_f| = n$. If w_i denotes the weight of K_i , where $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$. For any $a, b \geq 0$, and a, b are not 0 simultaneously. The ALCT-SFDWPPGHM operator is defined as follows:

$$ALCT - SFDWPPGHM^{a,b}(K_1, K_2, \dots, K_n) = \frac{1}{a+b} \left(\bigotimes_{f=1}^c \left(\bigotimes_{i=1}^{|O_f|} \bigotimes_{j=i}^{|O_f|} (aK_i)^{\frac{nw_{fi}(1+T(K_{fi}))}{\sum_{g=1}^n w_g(1+T(K_g))}} \oplus (bK_j)^{\frac{nw_{fj}(1+T(K_{fj}))}{\sum_{g=1}^n w_g(1+T(K_g))}} \right)^{\frac{2}{|O_f|(|O_f|+1)}} \right)^{\frac{1}{c}} \quad (34)$$

where $T(K_i) = \sum_{j=1, j \neq i}^n \text{Sup}(K_i, K_j)$, $\text{Sup}(K_i, K_j) = 1 - d(K_i, K_j)$, $\text{Sup}(K_i, K_j)$ indicate the support of K_j to K_i , which satisfies the following conditions:

- (1) $\text{Sup}(x_i, x_j) \in [0, 1]$;
- (2) $\text{Sup}(x_i, x_j) = \text{Sup}(x_j, x_i)$;
- (3) If $d(x_i, x_j) \leq d(K_m, K_p)$, then $\text{Sup}(x_i, x_j) \geq \text{Sup}(K_m, K_p)$.

Let

$$w'_i = \frac{1 + T(K_{fi})}{\sum_{i=1}^n (1 + T(K_g))}$$

where, $w'_i \in [0, 1]$, and $\sum_{i=1}^n w'_i = 1$, then

$$ALCT - SFDWPPGHM^{a,b}(K_1, K_2, \dots, K_n) = \frac{1}{a+b} \left(\bigotimes_{f=1}^c \left(\bigotimes_{i=1}^{|O_f|} \bigotimes_{j=i}^{|O_f|} (aK_i)^{\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w'_g w'_g}} \oplus (bK_j)^{\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w'_g w'_g}} \right)^{\frac{2}{|O_f|(|O_f|+1)}} \right)^{\frac{1}{c}} \quad (35)$$

Theorem 6. Let $K_i = (S_{\gamma_i}, S_{\delta_i}, S_{\tau_i}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs, divided into c partitions as $O_1, O_2, \dots, O_c, O_f = \{K_{f1}, K_{f2}, \dots, K_{f|O_f|}\} (f = 1, 2, \dots, c)$ and $\sum_{f=1}^c |O_f| = n$. If w_i denotes the weight of K_i , where $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$. For any $a, b \geq 0$, and a, b are not 0 simultaneously, $\lambda > 0$. Then, applying the ALCT-SFDWPPGGM operator to aggregate, the result is still a LCTSFN, which can be expressed as:

$$\begin{aligned}
 & \text{ALCT-SFDWPPGGM}^{a,b}(K_1, K_2, \dots, K_n) \\
 &= \left(S \sqrt[q]{\frac{t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\gamma_{fi})+b/\theta(\gamma_{fj})}))^{\frac{1}{\lambda}})}{1}} \right)^{\frac{1}{\lambda}} \\
 & \quad S \sqrt[q]{\frac{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\delta_{fi})+b/\theta(\delta_{fj})}))^{\frac{1}{\lambda}})}{1}} \right)^{\frac{1}{\lambda}} \\
 & \quad S \sqrt[q]{\frac{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})}))^{\frac{1}{\lambda}})}{1}} \right)^{\frac{1}{\lambda}} \\
 & \quad S \sqrt[q]{\frac{t^q - t^q / (1 + (\frac{1}{c} \sum_{f=1}^c (\frac{2(a+b)}{|O_f|(|O_f|+1)} \sum_{i=1}^{|O_f|} \sum_{j=i}^{|O_f|} \frac{1}{a/\theta(\tau_{fi})+b/\theta(\tau_{fj})}))^{\frac{1}{\lambda}})}{1}} \right)^{\frac{1}{\lambda}} \\
 & \text{where } g(h_{fi}) = \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fi})^q}{t^q - (\rho h_{fi})^q} \right)^{\lambda}, g(h_{fj}) = \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fj})^q}{t^q - (\rho h_{fj})^q} \right)^{\lambda}, \theta(h_{fi}) = \frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fi})^q}{(\rho h_{fi})^q} \right)^{\lambda}, \theta(h_{fj}) = \frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fj})^q}{(\rho h_{fj})^q} \right)^{\lambda}.
 \end{aligned} \tag{36}$$

The proof is similar to Theorem 2, so the proof is omitted here.

Theorem 7 (Idempotency). Let $K_i = (S_{\gamma_i}, S_{\delta_i}, S_{\tau_i}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs, where $a, b \geq 0$ and a, b are not 0 simultaneously.

For all $i = 1, 2, \dots, n$, if $K_i = K = (S_{\gamma}, S_{\delta}, S_{\tau})$, then

$$\text{ALCT-SFDWPPGGM}^{a,b}(K_1, K_2, \dots, K_n) = K \tag{37}$$

Theorem 8 (Monotonicity). Let $K_i = (S_{\gamma_i} e^{i2\pi S_{\gamma_i}}, S_{\delta_i} e^{i2\pi S_{\delta_i}}, S_{\tau_i} e^{i2\pi S_{\tau_i}}), K'_i = (S_{\gamma'_i} e^{i2\pi S_{\gamma'_i}}, S_{\delta'_i} e^{i2\pi S_{\delta'_i}}, S_{\tau'_i} e^{i2\pi S_{\tau'_i}}) (i = 1, 2, \dots, n)$ be two LCTSFNs, where $a, b \geq 0$ and a, b are not 0 simultaneously. For all $i = 1, 2, \dots, n$, If $S_{\gamma(f_i)} \leq S_{\gamma(f'_i)}, S_{\sigma_{\gamma(f_i)}} \leq S_{\sigma_{\gamma(f'_i)}}, S_{\delta(f_i)} \geq S_{\delta(f'_i)}, S_{\sigma_{\delta(f_i)}} \geq S_{\sigma_{\delta(f'_i)}}, S_{\tau(f_i)} \geq S_{\tau(f'_i)}, S_{\sigma_{\tau(f_i)}} \geq S_{\sigma_{\tau(f'_i)}}$, then

$$\text{ALCT-SFDWPPGGM}^{a,b}(K_1, K_2, \dots, K_n) \leq \text{ALCT-SFDWPPGGM}^{a,b}(K'_1, K'_2, \dots, K'_n) \tag{38}$$

Theorem 9 (Boundedness). Let $K_i = (S_{\gamma_i} e^{i2\pi S_{\gamma_i}}, S_{\delta_i} e^{i2\pi S_{\delta_i}}, S_{\tau_i} e^{i2\pi S_{\tau_i}}) (i = 1, 2, \dots, n)$ be a set of LCTSFNs, where $a, b \geq 0$ and a, b are not 0 simultaneously. $K^+ = (\max S_{\gamma_i} e^{i2\pi \max S_{\gamma_i}}, \min S_{\delta_i} e^{i2\pi \min S_{\delta_i}}, \min S_{\tau_i} e^{i2\pi \min S_{\tau_i}}), K^- = (\min S_{\gamma_i} e^{i2\pi \min S_{\gamma_i}}, \max S_{\delta_i} e^{i2\pi \max S_{\delta_i}}, \max S_{\tau_i} e^{i2\pi \max S_{\tau_i}})$, then

$$K^- \leq \text{ALCT-SFDWPPGGM}^{a,b}(K_1, K_2, \dots, K_n) \leq K^+ \tag{39}$$

The proofs of Theorems 7–9 are similar to those of Theorems 3–5, respectively, so the proofs are omitted here.

4. The Index System of the Quality Assessment of Emergency Information

Considering the characteristics of emergencies such as variability and no premonition [8], the release and acquisition of emergency information are crucial for the affected people and rescue personnel. The quality of emergency information is relevant to the achievement of the sustainable development goals and determines the efficiency of emer-

gency management. Accompanied by the rapid development of network technology, emergency information can be disseminated more quickly and widely through wireless networks in a short time. Some studies have shown that the attitude of social media users can indirectly reflect the quality of information on the media platform [51]. Therefore, this paper constructs an index system of the quality assessment of emergency information from the perspective of user's cognition and emotional experience. It is based on the communication characteristics of emergencies and the information features of online media, while adhering to scientific principles, comprehensive assessment standards, and objective practice. The assessment index system is divided into 4 dimensions with 16 indices in total, as shown in Table 1.

Table 1. The index system of the quality assessment of emergency information.

Name	Dimension	Index
The index system of the quality assessment of emergency information	Information source dimension (B ₁)	Source reliability (b ₁) Availability (b ₂) Security (b ₃)
	Information content dimension (B ₂)	Accuracy (b ₄) Integrity (b ₅) Rationalization (b ₆) Objectivity (b ₇)
	Information expression dimension (B ₃)	Comprehensibility (b ₈) Simplicity (b ₉) Standardization (b ₁₀) Innovativeness (b ₁₁)
	Information utility dimension (B ₄)	Timeliness (b ₁₂) Applicability (b ₁₃) Interactivity (b ₁₄) Usefulness (b ₁₅) Consistency (b ₁₆)

Information source dimension. The source of information greatly affects the quality of information, which can be divided into source reliability [52], availability [53], and security [54] indices. The source reliability index reflects the level of trust that information receiving groups have in the information they receive, which is often closely associated with the authority and reputation of users or institutions disseminating the information. The availability index is related to whether users can freely access the information on the online platform after emergencies occur. The degree of index is assessed based on the diversity and convenience of access channels. The security index manifests the degree of protection of emergency information. When the information dissemination platform is destroyed, it will face problems such as data leakage.

Information content dimension. Content is the most basic presentation of information and is a key factor in determining the quality of information. Information content dimension is categorized into accuracy [55], integrity [56], rationalization [57], and objectivity [58] indices. The accuracy index can reveal the degree of reality. The effective implementation of emergency decision-making must be based on detailed information content, which requires that the released information must accurately describe the causes and impacts of emergencies. The integrity index reflects the degree of completeness and comprehensiveness of emergency information description. Some scholars measure the grade of integrity by counting the cells filled with data [59]. The rationalization index is used to assess the credibility of information content. Supported by authoritative statements and high-quality evidence, the degree of rationalization will be higher. The objectivity index requires that the content of the information should not be biased and emotionally charged. The information compiler does not have personal subjective coloring, so as not to mislead the information receiver to make irrational judgments.

Information expression dimension. The information expression dimension should be assessed by taking into account the external form and expression mode of information, which consists of comprehensibility [60], simplicity [61], standardization [8], and innovativeness [8] indices. The comprehensibility index is used to measure the accessibility, comprehension, and usability of emergency information for decision-making by users. Emergency information needs to support readability for different audiences to meet users' needs for understanding and interpreting the information. The simplicity index reflects whether the statement of the released information is condensed, whether the logic is clear, and whether the form is concise. The development of the network era makes the information explode once an emergency occurs. Providing concise and focused emergency information can greatly shorten the time for users to retrieve information and improve the efficiency of emergency management. The standardization index represents whether the emergency information provided to users after an emergency is in line with the standard and logic. The innovativeness index shows the presence of diverse forms of information presentation, such as text, picture, video, audio, etc. Innovative forms of information can attract the attention of users to a great extent, which is more conducive to the diffusion and dissemination of emergency information.

Information utility dimension. The assessment of the utility of information reflects the extent to which emergency information meets the needs and expectations of a wide range of users and is a key aspect of information quality assessment. It is categorized into timeliness [62], applicability [63], interactivity [8], usefulness [54], and consistency [64] indices. The timeliness index is manifested in terms of the time span and value effect of information collection, organization, release, and transmission. As a result, emergency information for emergencies places higher demands on timeliness. The applicability index represents the level at which emergency information can meet the actual needs of users and be applied to specific practice. Therefore, the practical applicability should be fully considered before releasing the information. The interactivity index requires that when the network platform releases information to users, they can accept users' feedback and make timely adjustments in order to subsequently improve the quality of information. Practice shows that bidirectional communication is more conducive to decision-making adjustments by emergency management departments and improves decision-making efficiency. The usefulness index refers to the extent to which the information disseminated by online platforms effectively aids users in comprehending both the overall situation and specific details of an emergency, thereby reflecting the actual value and practical significance of emergency information for users. The consistency index embodies whether the grammar, identifications, and formats used between different data are consistent. The consistency of data information should be improved, so that users can better understand emergency information and better utilize its value.

5. Multi-Attribute Assessment Method Based on the ALCT-SFDWPPHM Operator Combined with the Entropy Measure and the WASPAS Method

Suppose $A = \{A_1, A_2, \dots, A_m\}$ represents m alternatives, $B = \{B_1, B_2, \dots, B_n\}$ represents n attributes, and the weight of attributes is denoted as $w = \{w_1, w_2, \dots, w_n\}$, respectively, where $w_i \in [0, 1]$, and $w_1 + w_2 + \dots + w_n = 1$. These attributes are divided into c partitions, denoted as O_1, O_2, \dots, O_c , and $|O_f| \in [1, n]$, $\sum_{f=1}^c |O_f| = n$. A group of experts $H = \{H_1, H_2, \dots, H_l\}$, with attribute weights $\xi = \{\xi_1, \xi_2, \dots, \xi_l\}$, where $\xi_i \in [0, 1]$, and $\xi_1 + \xi_2 + \dots + \xi_l = 1$. The linguistic complex T-spherical fuzzy assessment matrix denoted by $R = [K_{ij}]_{m \times n}$. $K_{ij} = \left(S_{\gamma_{ij}} e^{i2\pi S_{\sigma_{\gamma_{ij}}}}, S_{\delta_{ij}} e^{i2\pi S_{\sigma_{\delta_{ij}}}}, S_{\tau_{ij}} e^{i2\pi S_{\sigma_{\tau_{ij}}}} \right)$ shows the results of assessment with the attribute B_j of alternative A_i . $S_{\gamma_{ij}}$ and $S_{\sigma_{\gamma_{ij}}}$ indicate the degree of satisfaction with the attribute B_j of alternative A_i ; $S_{\tau_{ij}}$ and $S_{\sigma_{\tau_{ij}}}$ represent the degree of dissatisfaction with the attribute B_j of alternative A_i ; $S_{\delta_{ij}}$ and $S_{\sigma_{\delta_{ij}}}$ denote the degree of abstinence with the attribute B_j abstinence of alternative A_i . In order to cope with the situation where the weights of the attributes are unknown, the linguistic complex T-spherical

fuzzy information entropy measure method is proposed to solve this problem in this paper. Meanwhile, the proposed linguistic complex T-spherical fuzzy WASPAS method is used to solve the multi-attribute assessment problem with the following steps:

Step 1: Normalize assessment matrix. In real-world decision making, attributes are usually categorized into two types: cost attributes and benefit attributes, which have positive and negative effects on the aggregation results, respectively. In order to eliminate the influence of different attribute types, it is necessary to transform attributes into the same type. The rules for transforming the linguistic complex T-spherical fuzzy assessment matrix $R = [K_{ij}]_{m \times n}$ into a normalized linguistic complex T-spherical fuzzy assessment matrix are as follows:

$$K_{ij}' = \begin{cases} \left(S_{\gamma_{ij}} e^{i2\pi S_{\sigma_{\gamma_{ij}}}}, S_{\delta_{ij}} e^{i2\pi S_{\sigma_{\delta_{ij}}}}, S_{\tau_{ij}} e^{i2\pi S_{\sigma_{\tau_{ij}}}} \right), & \text{if } B_j \text{ is a benefit attribute} \\ \left(S_{\tau_{ij}} e^{i2\pi S_{\sigma_{\tau_{ij}}}}, S_{\delta_{ij}} e^{i2\pi S_{\sigma_{\delta_{ij}}}}, S_{\gamma_{ij}} e^{i2\pi S_{\sigma_{\gamma_{ij}}}} \right), & \text{if } B_j \text{ is a cost attribute} \end{cases} \quad (40)$$

Step 2: Apply the ALCT-SFDWPPHM operator to aggregate the assessment matrix of each expert into a collective assessment matrix.

$$K = ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \quad (41)$$

Step 3: Apply the linguistic complex T-spherical fuzzy entropy measure method to measure the unknown weights of the attributes based on the collective matrix obtained in step 2.

$$E_i = \frac{1}{h(2^{1/q} - 1)} \sum_{l=1}^h \left(S_{\gamma_i}^q + \sigma_{S_{\gamma_i}}^q + S_{\delta_i}^q + \sigma_{S_{\delta_i}}^q + S_{\tau_i}^q + \sigma_{S_{\tau_i}}^q \right) \quad (42)$$

where $\frac{1}{h(2^{1/q} - 1)}$ is a constant and $E_i \in [0, 1]$.

According to Equation (42), the weights of criteria are computed as follows:

$$w_i = \frac{1 - E_i}{h - \sum_{i=1}^h E_i} \quad (43)$$

Step 4: According to the ALCT-SFDWPPHM operator and ALCT-SFDWPPGHM operator, calculate the WSM and WPM of each alternative, and the results are expressed by $W_i^{(1)}$ and $W_i^{(2)}$, respectively.

$$W_i^{(1)} = ALCT - SFDWPPHM^{a,b}(K_1, K_2, \dots, K_n) \quad (44)$$

$$W_i^{(2)} = ALCT - SFDWPPGHM^{a,b}(K_1, K_2, \dots, K_n) \quad (45)$$

Step 5: Based on the $W_i^{(1)}$ and $W_i^{(2)}$ obtained in Step 4, the assessment results combined with the WASPAS method can be obtained as follows:

$$K = \beta W_i^{(1)} + (1 - \beta) W_i^{(2)} \quad (46)$$

Step 6: Calculate the score function of each alternative according to Equation (14); if the score values are the same, according to Equation (15), calculate its accuracy function again.

Step 7: Rank all the alternatives based on the score function and the accuracy function value to select the best alternative.

6. Numerical Example

In order to validate the effectiveness and applicability of the proposed multi-attribute assessment method, the emergency information quality assessment of four emergency information databases $\{A_1, A_2, A_3, A_4\}$ are planned, the results of which will be utilized in the rating process of the database. The assessment is mainly carried out in four dimensions

$\{B_1, B_2, B_3, B_4\}$, which contain a total of sixteen assessment attributes $\{b_1, b_2, b_3, \dots, b_{16}\}$, as shown in Table 1. In order to obtain more scientific and reasonable assessment results, three experts $\{H_1, H_2, H_3\}$ are invited to evaluate sixteen indices of four databases, and the obtained linguistic complex T-spherical fuzzy assessment matrices $R = [K_{ij}]_{m \times n}$ are shown in Tables 2–4, respectively. The relative importance level of the three experts is $\xi = \{0.25, 0.4, 0.35\}$.

Table 2. The linguistic complex T-spherical fuzzy assessment matrix R_1 given by H_1 .

	b_1	b_2	b_3	b_4
A_1	$(S_6 e^{i2\pi S_5}, S_5 e^{i2\pi S_2}, S_4 e^{i2\pi S_3})$	$(S_7 e^{i2\pi S_5}, S_4 e^{i2\pi S_4}, S_2 e^{i2\pi S_2})$	$(S_3 e^{i2\pi S_4}, S_4 e^{i2\pi S_6}, S_5 e^{i2\pi S_2})$	$(S_1 e^{i2\pi S_5}, S_4 e^{i2\pi S_5}, S_7 e^{i2\pi S_6})$
A_2	$(S_4 e^{i2\pi S_5}, S_7 e^{i2\pi S_7}, S_2 e^{i2\pi S_1})$	$(S_7 e^{i2\pi S_5}, S_2 e^{i2\pi S_3}, S_5 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_3}, S_5 e^{i2\pi S_7}, S_3 e^{i2\pi S_5})$	$(S_3 e^{i2\pi S_4}, S_3 e^{i2\pi S_2}, S_2 e^{i2\pi S_3})$
A_3	$(S_2 e^{i2\pi S_3}, S_6 e^{i2\pi S_5}, S_3 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_6}, S_2 e^{i2\pi S_5}, S_3 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_7}, S_6 e^{i2\pi S_4}, S_5 e^{i2\pi S_3})$	$(S_4 e^{i2\pi S_7}, S_6 e^{i2\pi S_5}, S_4 e^{i2\pi S_3})$
A_4	$(S_7 e^{i2\pi S_5}, S_4 e^{i2\pi S_3}, S_3 e^{i2\pi S_5})$	$(S_2 e^{i2\pi S_2}, S_5 e^{i2\pi S_4}, S_3 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_5}, S_3 e^{i2\pi S_3}, S_6 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_2}, S_7 e^{i2\pi S_3}, S_2 e^{i2\pi S_4})$
	b_5	b_6	b_7	b_8
A_1	$(S_5 e^{i2\pi S_5}, S_2 e^{i2\pi S_1}, S_3 e^{i2\pi S_6})$	$(S_4 e^{i2\pi S_4}, S_5 e^{i2\pi S_6}, S_4 e^{i2\pi S_2})$	$(S_2 e^{i2\pi S_7}, S_3 e^{i2\pi S_2}, S_5 e^{i2\pi S_3})$	$(S_4 e^{i2\pi S_4}, S_6 e^{i2\pi S_1}, S_1 e^{i2\pi S_7})$
A_2	$(S_6 e^{i2\pi S_5}, S_4 e^{i2\pi S_2}, S_5 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_3}, S_1 e^{i2\pi S_2}, S_2 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_2}, S_3 e^{i2\pi S_2}, S_7 e^{i2\pi S_6})$	$(S_3 e^{i2\pi S_5}, S_7 e^{i2\pi S_2}, S_4 e^{i2\pi S_3})$
A_3	$(S_5 e^{i2\pi S_4}, S_2 e^{i2\pi S_4}, S_1 e^{i2\pi S_3})$	$(S_7 e^{i2\pi S_2}, S_1 e^{i2\pi S_4}, S_3 e^{i2\pi S_6})$	$(S_7 e^{i2\pi S_1}, S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_6}, S_1 e^{i2\pi S_5}, S_2 e^{i2\pi S_5})$
A_4	$(S_7 e^{i2\pi S_6}, S_3 e^{i2\pi S_2}, S_4 e^{i2\pi S_2})$	$(S_5 e^{i2\pi S_4}, S_3 e^{i2\pi S_5}, S_7 e^{i2\pi S_1})$	$(S_2 e^{i2\pi S_3}, S_3 e^{i2\pi S_5}, S_6 e^{i2\pi S_6})$	$(S_4 e^{i2\pi S_2}, S_4 e^{i2\pi S_3}, S_7 e^{i2\pi S_4})$
	b_9	b_{10}	b_{11}	b_{12}
A_1	$(S_2 e^{i2\pi S_6}, S_7 e^{i2\pi S_4}, S_2 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_7}, S_2 e^{i2\pi S_4}, S_6 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_7}, S_1 e^{i2\pi S_2}, S_2 e^{i2\pi S_3})$	$(S_5 e^{i2\pi S_4}, S_5 e^{i2\pi S_6}, S_6 e^{i2\pi S_3})$
A_2	$(S_7 e^{i2\pi S_3}, S_2 e^{i2\pi S_6}, S_5 e^{i2\pi S_2})$	$(S_2 e^{i2\pi S_7}, S_2 e^{i2\pi S_5}, S_7 e^{i2\pi S_3})$	$(S_5 e^{i2\pi S_3}, S_5 e^{i2\pi S_5}, S_1 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_3}, S_2 e^{i2\pi S_2}, S_4 e^{i2\pi S_3})$
A_3	$(S_7 e^{i2\pi S_1}, S_2 e^{i2\pi S_7}, S_5 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_3}, S_6 e^{i2\pi S_5}, S_4 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_6}, S_6 e^{i2\pi S_5})$	$(S_1 e^{i2\pi S_5}, S_7 e^{i2\pi S_3}, S_5 e^{i2\pi S_5})$
A_4	$(S_5 e^{i2\pi S_3}, S_3 e^{i2\pi S_5}, S_1 e^{i2\pi S_4})$	$(S_4 e^{i2\pi S_4}, S_5 e^{i2\pi S_4}, S_6 e^{i2\pi S_6})$	$(S_1 e^{i2\pi S_4}, S_2 e^{i2\pi S_5}, S_4 e^{i2\pi S_6})$	$(S_3 e^{i2\pi S_5}, S_3 e^{i2\pi S_3}, S_7 e^{i2\pi S_4})$
	b_{13}	b_{14}	b_{15}	b_{16}
A_1	$(S_2 e^{i2\pi S_4}, S_3 e^{i2\pi S_4}, S_3 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_4}, S_6 e^{i2\pi S_5}, S_4 e^{i2\pi S_1})$	$(S_4 e^{i2\pi S_4}, S_4 e^{i2\pi S_2}, S_5 e^{i2\pi S_5})$	$(S_3 e^{i2\pi S_7}, S_5 e^{i2\pi S_1}, S_5 e^{i2\pi S_2})$
A_2	$(S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_5}, S_2 e^{i2\pi S_6})$	$(S_6 e^{i2\pi S_2}, S_3 e^{i2\pi S_7}, S_5 e^{i2\pi S_1})$	$(S_7 e^{i2\pi S_2}, S_2 e^{i2\pi S_2}, S_4 e^{i2\pi S_6})$	$(S_2 e^{i2\pi S_2}, S_5 e^{i2\pi S_4}, S_7 e^{i2\pi S_4})$
A_3	$(S_4 e^{i2\pi S_5}, S_4 e^{i2\pi S_3}, S_3 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_5}, S_5 e^{i2\pi S_1}, S_3 e^{i2\pi S_5})$	$(S_3 e^{i2\pi S_5}, S_5 e^{i2\pi S_3}, S_5 e^{i2\pi S_6})$	$(S_2 e^{i2\pi S_4}, S_7 e^{i2\pi S_2}, S_3 e^{i2\pi S_6})$
A_4	$(S_3 e^{i2\pi S_5}, S_5 e^{i2\pi S_2}, S_6 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_6}, S_5 e^{i2\pi S_2}, S_3 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_5}, S_6 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_5}, S_5 e^{i2\pi S_2}, S_6 e^{i2\pi S_2})$

Table 3. The linguistic complex T-spherical fuzzy assessment matrix R_2 given by H_2 .

	b_1	b_2	b_3	b_4
A_1	$(S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_2}, S_3 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_5}, S_6 e^{i2\pi S_3}, S_3 e^{i2\pi S_4})$	$(S_4 e^{i2\pi S_5}, S_5 e^{i2\pi S_1}, S_6 e^{i2\pi S_3})$	$(S_2 e^{i2\pi S_4}, S_3 e^{i2\pi S_5}, S_6 e^{i2\pi S_5})$
A_2	$(S_5 e^{i2\pi S_5}, S_6 e^{i2\pi S_5}, S_2 e^{i2\pi S_2})$	$(S_7 e^{i2\pi S_5}, S_3 e^{i2\pi S_4}, S_5 e^{i2\pi S_3})$	$(S_5 e^{i2\pi S_5}, S_6 e^{i2\pi S_5}, S_3 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_4}, S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_3})$
A_3	$(S_3 e^{i2\pi S_3}, S_7 e^{i2\pi S_4}, S_4 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_5}, S_3 e^{i2\pi S_4}, S_4 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_4}, S_5 e^{i2\pi S_3}, S_2 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_5}, S_3 e^{i2\pi S_2}, S_4 e^{i2\pi S_3})$
A_4	$(S_6 e^{i2\pi S_4}, S_7 e^{i2\pi S_3}, S_2 e^{i2\pi S_4})$	$(S_1 e^{i2\pi S_2}, S_2 e^{i2\pi S_4}, S_3 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_5}, S_3 e^{i2\pi S_4}, S_5 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_4}, S_5 e^{i2\pi S_2}, S_3 e^{i2\pi S_5})$
	b_5	b_6	b_7	b_8
A_1	$(S_3 e^{i2\pi S_2}, S_6 e^{i2\pi S_1}, S_4 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_6}, S_4 e^{i2\pi S_4}, S_6 e^{i2\pi S_5})$	$(S_3 e^{i2\pi S_3}, S_2 e^{i2\pi S_2}, S_5 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_6}, S_3 e^{i2\pi S_3}, S_4 e^{i2\pi S_2})$
A_2	$(S_6 e^{i2\pi S_5}, S_5 e^{i2\pi S_4}, S_4 e^{i2\pi S_1})$	$(S_6 e^{i2\pi S_3}, S_3 e^{i2\pi S_6}, S_5 e^{i2\pi S_4})$	$(S_4 e^{i2\pi S_5}, S_4 e^{i2\pi S_4}, S_7 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_4}, S_1 e^{i2\pi S_6}, S_3 e^{i2\pi S_6})$
A_3	$(S_7 e^{i2\pi S_6}, S_1 e^{i2\pi S_3}, S_4 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_7}, S_2 e^{i2\pi S_1}, S_2 e^{i2\pi S_2})$	$(S_3 e^{i2\pi S_6}, S_1 e^{i2\pi S_6}, S_1 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_3}, S_3 e^{i2\pi S_3}, S_2 e^{i2\pi S_4})$
A_4	$(S_4 e^{i2\pi S_4}, S_5 e^{i2\pi S_2}, S_5 e^{i2\pi S_3})$	$(S_6 e^{i2\pi S_2}, S_4 e^{i2\pi S_4}, S_3 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_4}, S_6 e^{i2\pi S_2}, S_5 e^{i2\pi S_3})$	$(S_3 e^{i2\pi S_5}, S_4 e^{i2\pi S_2}, S_7 e^{i2\pi S_6})$
	b_9	b_{10}	b_{11}	b_{12}
A_1	$(S_3 e^{i2\pi S_4}, S_5 e^{i2\pi S_5}, S_3 e^{i2\pi S_3})$	$(S_5 e^{i2\pi S_6}, S_3 e^{i2\pi S_1}, S_5 e^{i2\pi S_3})$	$(S_7 e^{i2\pi S_4}, S_2 e^{i2\pi S_4}, S_2 e^{i2\pi S_5})$	$(S_6 e^{i2\pi S_5}, S_2 e^{i2\pi S_2}, S_5 e^{i2\pi S_4})$
A_2	$(S_4 e^{i2\pi S_1}, S_5 e^{i2\pi S_5}, S_2 e^{i2\pi S_3})$	$(S_4 e^{i2\pi S_5}, S_4 e^{i2\pi S_4}, S_5 e^{i2\pi S_4})$	$(S_4 e^{i2\pi S_5}, S_5 e^{i2\pi S_3}, S_3 e^{i2\pi S_7})$	$(S_3 e^{i2\pi S_6}, S_4 e^{i2\pi S_2}, S_2 e^{i2\pi S_5})$
A_3	$(S_7 e^{i2\pi S_2}, S_5 e^{i2\pi S_6}, S_2 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_4}, S_6 e^{i2\pi S_4}, S_4 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_4}, S_5 e^{i2\pi S_4}, S_4 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_3}, S_6 e^{i2\pi S_5}, S_6 e^{i2\pi S_6})$
A_4	$(S_4 e^{i2\pi S_3}, S_5 e^{i2\pi S_7}, S_2 e^{i2\pi S_3})$	$(S_3 e^{i2\pi S_5}, S_6 e^{i2\pi S_4}, S_4 e^{i2\pi S_3})$	$(S_2 e^{i2\pi S_4}, S_2 e^{i2\pi S_4}, S_5 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_4}, S_4 e^{i2\pi S_4}, S_3 e^{i2\pi S_3})$
	b_{13}	b_{14}	b_{15}	b_{16}
A_1	$(S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_5}, S_5 e^{i2\pi S_4})$	$(S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_4}, S_7 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_6}, S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_5}, S_6 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$
A_2	$(S_5 e^{i2\pi S_4}, S_4 e^{i2\pi S_6}, S_3 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_5}, S_4 e^{i2\pi S_2}, S_6 e^{i2\pi S_5})$	$(S_6 e^{i2\pi S_6}, S_4 e^{i2\pi S_4}, S_2 e^{i2\pi S_5})$	$(S_3 e^{i2\pi S_4}, S_6 e^{i2\pi S_2}, S_6 e^{i2\pi S_7})$
A_3	$(S_2 e^{i2\pi S_4}, S_5 e^{i2\pi S_4}, S_5 e^{i2\pi S_5})$	$(S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_5}, S_6 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_6}, S_5 e^{i2\pi S_4}, S_6 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_3}, S_6 e^{i2\pi S_3}, S_4 e^{i2\pi S_6})$
A_4	$(S_5 e^{i2\pi S_5}, S_4 e^{i2\pi S_3}, S_5 e^{i2\pi S_3})$	$(S_5 e^{i2\pi S_4}, S_7 e^{i2\pi S_5}, S_2 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_5}, S_3 e^{i2\pi S_3}, S_5 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_7}, S_2 e^{i2\pi S_1}, S_4 e^{i2\pi S_2})$

Table 4. The linguistic complex T-spherical fuzzy assessment matrix R_3 given by H_3 .

	b_1	b_2	b_3	b_4
A_1	$(S_5 e^{i2\pi S_4}, S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_6}, S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_4}, S_7 e^{i2\pi S_4}, S_2 e^{i2\pi S_3})$	$(S_1 e^{i2\pi S_3}, S_5 e^{i2\pi S_6}, S_7 e^{i2\pi S_4})$
A_2	$(S_6 e^{i2\pi S_3}, S_4 e^{i2\pi S_3}, S_3 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_6}, S_4 e^{i2\pi S_3}, S_3 e^{i2\pi S_3})$	$(S_4 e^{i2\pi S_5}, S_3 e^{i2\pi S_4}, S_2 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_5}, S_3 e^{i2\pi S_2}, S_4 e^{i2\pi S_4})$
A_3	$(S_5 e^{i2\pi S_6}, S_6 e^{i2\pi S_4}, S_4 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_4}, S_3 e^{i2\pi S_4}, S_4 e^{i2\pi S_3})$	$(S_3 e^{i2\pi S_5}, S_4 e^{i2\pi S_4}, S_2 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_2}, S_2 e^{i2\pi S_3}, S_6 e^{i2\pi S_4})$
A_4	$(S_5 e^{i2\pi S_5}, S_3 e^{i2\pi S_4}, S_3 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_4}, S_7 e^{i2\pi S_3}, S_1 e^{i2\pi S_6})$	$(S_4 e^{i2\pi S_6}, S_4 e^{i2\pi S_3}, S_5 e^{i2\pi S_5})$
	b_5	b_6	b_7	b_8
A_1	$(S_6 e^{i2\pi S_4}, S_1 e^{i2\pi S_2}, S_2 e^{i2\pi S_1})$	$(S_4 e^{i2\pi S_6}, S_1 e^{i2\pi S_6}, S_7 e^{i2\pi S_2})$	$(S_2 e^{i2\pi S_5}, S_6 e^{i2\pi S_3}, S_3 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_2}, S_2 e^{i2\pi S_4}, S_6 e^{i2\pi S_1})$
A_2	$(S_4 e^{i2\pi S_3}, S_3 e^{i2\pi S_6}, S_2 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_6}, S_4 e^{i2\pi S_5}, S_3 e^{i2\pi S_3})$	$(S_2 e^{i2\pi S_3}, S_5 e^{i2\pi S_7}, S_4 e^{i2\pi S_5})$	$(S_5 e^{i2\pi S_3}, S_1 e^{i2\pi S_4}, S_5 e^{i2\pi S_2})$
A_3	$(S_7 e^{i2\pi S_5}, S_1 e^{i2\pi S_4}, S_3 e^{i2\pi S_2})$	$(S_4 e^{i2\pi S_5}, S_3 e^{i2\pi S_2}, S_2 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_3}, S_3 e^{i2\pi S_5}, S_5 e^{i2\pi S_6})$	$(S_3 e^{i2\pi S_4}, S_1 e^{i2\pi S_2}, S_1 e^{i2\pi S_4})$
A_4	$(S_5 e^{i2\pi S_4}, S_4 e^{i2\pi S_6}, S_5 e^{i2\pi S_3})$	$(S_3 e^{i2\pi S_2}, S_2 e^{i2\pi S_5}, S_7 e^{i2\pi S_5})$	$(S_1 e^{i2\pi S_2}, S_3 e^{i2\pi S_6}, S_7 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_6}, S_1 e^{i2\pi S_2}, S_3 e^{i2\pi S_6})$
	b_9	b_{10}	b_{11}	b_{12}
A_1	$(S_5 e^{i2\pi S_3}, S_5 e^{i2\pi S_6}, S_4 e^{i2\pi S_4})$	$(S_2 e^{i2\pi S_5}, S_6 e^{i2\pi S_2}, S_4 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_5}, S_4 e^{i2\pi S_4}, S_4 e^{i2\pi S_3})$	$(S_5 e^{i2\pi S_6}, S_4 e^{i2\pi S_3}, S_6 e^{i2\pi S_1})$
A_2	$(S_6 e^{i2\pi S_2}, S_3 e^{i2\pi S_7}, S_2 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_6}, S_2 e^{i2\pi S_4}, S_6 e^{i2\pi S_2})$	$(S_3 e^{i2\pi S_4}, S_3 e^{i2\pi S_4}, S_4 e^{i2\pi S_6})$	$(S_4 e^{i2\pi S_6}, S_5 e^{i2\pi S_3}, S_6 e^{i2\pi S_1})$
A_3	$(S_5 e^{i2\pi S_4}, S_2 e^{i2\pi S_7}, S_3 e^{i2\pi S_5})$	$(S_4 e^{i2\pi S_3}, S_4 e^{i2\pi S_5}, S_5 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_5}, S_2 e^{i2\pi S_6}, S_3 e^{i2\pi S_4})$	$(S_3 e^{i2\pi S_5}, S_5 e^{i2\pi S_4}, S_5 e^{i2\pi S_5})$
A_4	$(S_6 e^{i2\pi S_4}, S_2 e^{i2\pi S_6}, S_2 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_3}, S_5 e^{i2\pi S_5}, S_6 e^{i2\pi S_3})$	$(S_4 e^{i2\pi S_5}, S_4 e^{i2\pi S_2}, S_6 e^{i2\pi S_4})$	$(S_7 e^{i2\pi S_5}, S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$
	b_{13}	b_{14}	b_{15}	b_{16}
A_1	$(S_3 e^{i2\pi S_5}, S_2 e^{i2\pi S_3}, S_4 e^{i2\pi S_3})$	$(S_6 e^{i2\pi S_4}, S_4 e^{i2\pi S_4}, S_5 e^{i2\pi S_2})$	$(S_6 e^{i2\pi S_5}, S_3 e^{i2\pi S_4}, S_2 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_6}, S_5 e^{i2\pi S_4}, S_6 e^{i2\pi S_3})$
A_2	$(S_6 e^{i2\pi S_5}, S_5 e^{i2\pi S_3}, S_3 e^{i2\pi S_6})$	$(S_5 e^{i2\pi S_4}, S_5 e^{i2\pi S_5}, S_4 e^{i2\pi S_3})$	$(S_6 e^{i2\pi S_5}, S_3 e^{i2\pi S_3}, S_4 e^{i2\pi S_6})$	$(S_4 e^{i2\pi S_5}, S_7 e^{i2\pi S_3}, S_5 e^{i2\pi S_5})$
A_3	$(S_1 e^{i2\pi S_5}, S_6 e^{i2\pi S_3}, S_4 e^{i2\pi S_4})$	$(S_5 e^{i2\pi S_4}, S_3 e^{i2\pi S_4}, S_4 e^{i2\pi S_6})$	$(S_2 e^{i2\pi S_5}, S_4 e^{i2\pi S_6}, S_5 e^{i2\pi S_6})$	$(S_3 e^{i2\pi S_2}, S_3 e^{i2\pi S_3}, S_5 e^{i2\pi S_3})$
A_4	$(S_6 e^{i2\pi S_7}, S_3 e^{i2\pi S_1}, S_4 e^{i2\pi S_4})$	$(S_7 e^{i2\pi S_5}, S_2 e^{i2\pi S_3}, S_2 e^{i2\pi S_4})$	$(S_6 e^{i2\pi S_2}, S_4 e^{i2\pi S_5}, S_5 e^{i2\pi S_4})$	$(S_4 e^{i2\pi S_5}, S_3 e^{i2\pi S_3}, S_5 e^{i2\pi S_4})$

6.1. Assessment Ranking

Step 1: Normalize the assessment matrix. This step can be ignored because all attributes are benefit attributes. The normalized assessment matrices are still shown in Tables 2–4.

Step 2: Aggregate each experts' assessment matrix into a collective assessment matrix according to Equation (41), as shown in Table 5.

Table 5. The collective linguistic complex T-spherical fuzzy assessment matrix.

	b_1	b_2
A_1	$(S_{5.2339} e^{i2\pi S_{4.2483}}, S_{4.1273} e^{i2\pi S_{2.1505}}, S_{3.4086} e^{i2\pi S_{3.5837}})$	$(S_{6.4020} e^{i2\pi S_{5.6164}}, S_{4.2930} e^{i2\pi S_{3.1284}}, S_{2.4826} e^{i2\pi S_{2.5017}})$
A_2	$(S_{5.5083} e^{i2\pi S_{4.7413}}, S_{4.6890} e^{i2\pi S_{3.5898}}, S_{2.1343} e^{i2\pi S_{1.2661}})$	$(S_{6.8339} e^{i2\pi S_{5.6103}}, S_{2.4776} e^{i2\pi S_{3.1905}}, S_{3.5732} e^{i2\pi S_{2.4572}})$
A_3	$(S_{4.2891} e^{i2\pi S_{5.2729}}, S_{6.2287} e^{i2\pi S_{4.1350}}, S_{3.5801} e^{i2\pi S_{4.6361}})$	$(S_{5.3503} e^{i2\pi S_{5.2477}}, S_{2.4754} e^{i2\pi S_{4.1309}}, S_{3.5910} e^{i2\pi S_{3.3568}})$
A_4	$(S_{6.3868} e^{i2\pi S_{4.7859}}, S_{2.3654} e^{i2\pi S_{3.1961}}, S_{2.3316} e^{i2\pi S_{4.1299}})$	$(S_{1.8420} e^{i2\pi S_{2.5915}}, S_{2.3819} e^{i2\pi S_{3.3581}}, S_{3.1940} e^{i2\pi S_{4.3780}})$
	b_3	b_4
A_1	$(S_{4.4953} e^{i2\pi S_{4.5484}}, S_{4.7524} e^{i2\pi S_{1.1900}}, S_{2.4215} e^{i2\pi S_{2.4726}})$	$(S_{1.6791} e^{i2\pi S_{4.2498}}, S_{3.4382} e^{i2\pi S_{5.2426}}, S_{6.4649} e^{i2\pi S_{4.4749}})$
A_2	$(S_{5.2503} e^{i2\pi S_{4.7605}}, S_{3.5898} e^{i2\pi S_{4.5048}}, S_{2.3274} e^{i2\pi S_{4.1343}})$	$(S_{5.2646} e^{i2\pi S_{4.5939}}, S_{3.1895} e^{i2\pi S_{2.1549}}, S_{2.5004} e^{i2\pi S_{3.1870}})$
A_3	$(S_{4.0806} e^{i2\pi S_{6.2802}}, S_{4.4553} e^{i2\pi S_{3.3997}}, S_{2.1215} e^{i2\pi S_{3.6128}})$	$(S_{4.8612} e^{i2\pi S_{6.2969}}, S_{2.3544} e^{i2\pi S_{2.3567}}, S_{4.2563} e^{i2\pi S_{3.1919}})$
A_4	$(S_{4.8509} e^{i2\pi S_{4.7693}}, S_{3.2040} e^{i2\pi S_{3.1912}}, S_{1.2225} e^{i2\pi S_{5.4485}})$	$(S_{4.3297} e^{i2\pi S_{5.3162}}, S_{4.4998} e^{i2\pi S_{2.3210}}, S_{2.5062} e^{i2\pi S_{4.6517}})$
	b_5	b_6
A_1	$(S_{5.4653} e^{i2\pi S_{4.2240}}, S_{1.2051} e^{i2\pi S_{1.0790}}, S_{2.3632} e^{i2\pi S_{1.2071}})$	$(S_{4.5410} e^{i2\pi S_{5.7851}}, S_{1.2090} e^{i2\pi S_{4.6196}}, S_{4.8634} e^{i2\pi S_{2.1715}})$
A_2	$(S_{5.7595} e^{i2\pi S_{4.7458}}, S_{3.4874} e^{i2\pi S_{2.4898}}, S_{2.4425} e^{i2\pi S_{1.1877}})$	$(S_{5.4085} e^{i2\pi S_{5.2990}}, S_{1.2658} e^{i2\pi S_{2.5200}}, S_{2.5003} e^{i2\pi S_{3.4215}})$
A_3	$(S_{6.8461} e^{i2\pi S_{5.4206}}, S_{1.0632} e^{i2\pi S_{3.4150}}, S_{1.2647} e^{i2\pi S_{2.3584}})$	$(S_{6.3069} e^{i2\pi S_{6.5092}}, S_{1.2711} e^{i2\pi S_{1.1928}}, S_{2.1056} e^{i2\pi S_{2.3785}})$
A_4	$(S_{6.3204} e^{i2\pi S_{5.1430}}, S_{3.6538} e^{i2\pi S_{2.1508}}, S_{4.6452} e^{i2\pi S_{2.4750}})$	$(S_{5.4825} e^{i2\pi S_{3.2498}}, S_{2.3629} e^{i2\pi S_{4.4471}}, S_{3.5612} e^{i2\pi S_{1.2572}})$

Table 5. Cont.

b₇			b₈		
A ₁	$(S_{2.5844}e^{i2\pi S_{6.3332}}, S_{2.3425}e^{i2\pi S_{2.1427}}, S_{3.5566}e^{i2\pi S_{3.5821}})$		$(S_{5.3784}e^{i2\pi S_{5.3779}}, S_{2.3573}e^{i2\pi S_{1.2686}}, S_{1.2687}e^{i2\pi S_{1.2100}})$		
A ₂	$(S_{4.2700}e^{i2\pi S_{4.3711}}, S_{3.6197}e^{i2\pi S_{2.4894}}, S_{4.7781}e^{i2\pi S_{4.5349}})$		$(S_{4.3498}e^{i2\pi S_{4.2410}}, S_{1.0651}e^{i2\pi S_{2.5075}}, S_{3.4608}e^{i2\pi S_{2.3564}})$		
A ₃	$(S_{6.3988}e^{i2\pi S_{5.2855}}, S_{1.2101}e^{i2\pi S_{3.6994}}, S_{1.2123}e^{i2\pi S_{2.4159}})$		$(S_{5.4952}e^{i2\pi S_{5.1266}}, S_{1.0876}e^{i2\pi S_{2.3518}}, S_{1.2040}e^{i2\pi S_{4.1284}})$		
A ₄	$(S_{3.3840}e^{i2\pi S_{3.4896}}, S_{3.2319}e^{i2\pi S_{2.3999}}, S_{5.5375}e^{i2\pi S_{3.5248}})$		$(S_{3.3037}e^{i2\pi S_{5.4660}}, S_{1.2302}e^{i2\pi S_{2.1172}}, S_{3.6395}e^{i2\pi S_{4.8379}})$		
b₉			b₁₀		
A ₁	$(S_{4.3026}e^{i2\pi S_{5.1340}}, S_{5.2379}e^{i2\pi S_{4.7428}}, S_{2.4959}e^{i2\pi S_{2.4959}})$		$(S_{5.2331}e^{i2\pi S_{6.3780}}, S_{2.5054}e^{i2\pi S_{1.1847}}, S_{4.4806}e^{i2\pi S_{2.5038}})$		
A ₂	$(S_{6.3830}e^{i2\pi S_{2.4151}}, S_{2.4930}e^{i2\pi S_{5.5242}}, S_{2.1313}e^{i2\pi S_{2.4903}})$		$(S_{4.4951}e^{i2\pi S_{6.3887}}, S_{2.1659}e^{i2\pi S_{4.1263}}, S_{5.5969}e^{i2\pi S_{2.3619}})$		
A ₃	$(S_{6.8116}e^{i2\pi S_{3.3549}}, S_{2.1716}e^{i2\pi S_{6.4666}}, S_{2.3767}e^{i2\pi S_{4.5661}})$		$(S_{4.2342}e^{i2\pi S_{3.5457}}, S_{4.6434}e^{i2\pi S_{4.4384}}, S_{4.2275}e^{i2\pi S_{4.2009}})$		
A ₄	$(S_{5.4873}e^{i2\pi S_{3.5973}}, S_{2.3715}e^{i2\pi S_{5.7509}}, S_{1.2568}e^{i2\pi S_{3.4060}})$		$(S_{4.4617}e^{i2\pi S_{4.4774}}, S_{5.2097}e^{i2\pi S_{4.2314}}, S_{4.6294}e^{i2\pi S_{3.1909}})$		
b₁₁			b₁₂		
A ₁	$(S_{6.5907}e^{i2\pi S_{6.3341}}, S_{1.2608}e^{i2\pi S_{2.5092}}, S_{2.1529}e^{i2\pi S_{3.2350}})$		$(S_{5.5547}e^{i2\pi S_{5.5309}}, S_{2.3868}e^{i2\pi S_{2.3764}}, S_{5.4632}e^{i2\pi S_{1.2079}})$		
A ₂	$(S_{4.2404}e^{i2\pi S_{4.4111}}, S_{3.5467}e^{i2\pi S_{3.4889}}, S_{1.2631}e^{i2\pi S_{4.8503}})$		$(S_{3.5089}e^{i2\pi S_{5.7615}}, S_{2.4997}e^{i2\pi S_{2.1471}}, S_{2.3868}e^{i2\pi S_{1.2115}})$		
A ₃	$(S_{5.4939}e^{i2\pi S_{4.4953}}, S_{2.4158}e^{i2\pi S_{4.6221}}, S_{3.4446}e^{i2\pi S_{4.3769}})$		$(S_{3.4320}e^{i2\pi S_{4.6978}}, S_{5.5042}e^{i2\pi S_{3.6311}}, S_{5.2100}e^{i2\pi S_{5.2100}})$		
A ₄	$(S_{3.3518}e^{i2\pi S_{4.5972}}, S_{2.1441}e^{i2\pi S_{2.4210}}, S_{4.7397}e^{i2\pi S_{2.3687}})$		$(S_{6.5942}e^{i2\pi S_{4.7899}}, S_{2.3660}e^{i2\pi S_{3.1776}}, S_{3.5093}e^{i2\pi S_{3.4050}})$		
b₁₃			b₁₄		
A ₁	$(S_{3.4416}e^{i2\pi S_{4.4624}}, S_{2.3668}e^{i2\pi S_{3.4552}}, S_{3.6406}e^{i2\pi S_{3.4372}})$		$(S_{5.4915}e^{i2\pi S_{3.7920}}, S_{4.2162}e^{i2\pi S_{4.1289}}, S_{4.7504}e^{i2\pi S_{1.2590}})$		
A ₂	$(S_{5.5131}e^{i2\pi S_{4.4886}}, S_{4.2223}e^{i2\pi S_{3.5868}}, S_{2.4589}e^{i2\pi S_{5.4540}})$		$(S_{5.3560}e^{i2\pi S_{4.3887}}, S_{3.6533}e^{i2\pi S_{2.3850}}, S_{4.5073}e^{i2\pi S_{1.2638}})$		
A ₃	$(S_{3.2402}e^{i2\pi S_{4.7883}}, S_{4.7348}e^{i2\pi S_{3.1789}}, S_{3.6401}e^{i2\pi S_{4.1982}})$		$(S_{4.6892}e^{i2\pi S_{4.2492}}, S_{3.4203}e^{i2\pi S_{1.2622}}, S_{3.6597}e^{i2\pi S_{5.6580}})$		
A ₄	$(S_{5.4862}e^{i2\pi S_{6.4746}}, S_{3.4383}e^{i2\pi S_{1.2174}}, S_{4.4856}e^{i2\pi S_{3.4773}})$		$(S_{6.4744}e^{i2\pi S_{5.2757}}, S_{2.4194}e^{i2\pi S_{2.4835}}, S_{2.1120}e^{i2\pi S_{4.3771}})$		
b₁₅			b₁₆		
A ₁	$(S_{5.5188}e^{i2\pi S_{5.4391}}, S_{2.3643}e^{i2\pi S_{2.4873}}, S_{2.4158}e^{i2\pi S_{4.4893}})$		$(S_{4.3160}e^{i2\pi S_{6.4076}}, S_{5.2143}e^{i2\pi S_{1.2563}}, S_{4.4940}e^{i2\pi S_{2.4799}})$		
A ₂	$(S_{6.4437}e^{i2\pi S_{5.4000}}, S_{2.4962}e^{i2\pi S_{2.4962}}, S_{2.3842}e^{i2\pi S_{5.4642}})$		$(S_{3.5109}e^{i2\pi S_{4.4836}}, S_{5.7631}e^{i2\pi S_{2.3666}}, S_{5.5022}e^{i2\pi S_{4.7594}})$		
A ₃	$(S_{4.3312}e^{i2\pi S_{5.5548}}, S_{4.3778}e^{i2\pi S_{3.6301}}, S_{5.2120}e^{i2\pi S_{5.4631}})$		$(S_{4.3578}e^{i2\pi S_{3.3186}}, S_{3.6290}e^{i2\pi S_{2.4561}}, S_{3.6293}e^{i2\pi S_{3.6272}})$		
A ₄	$(S_{5.3586}e^{i2\pi S_{4.3199}}, S_{3.4131}e^{i2\pi S_{3.5471}}, S_{5.1401}e^{i2\pi S_{4.2723}})$		$(S_{4.4013}e^{i2\pi S_{6.5043}}, S_{2.3804}e^{i2\pi S_{1.1983}}, S_{4.5696}e^{i2\pi S_{2.1578}})$		

Step 3: The attribute weights are obtained based on Equations (42) and (43) as shown in Table 6.

Table 6. Attribute weights.

Dimension	Weight	Attribute	Entropy Weight
B ₁	0.1869	b ₁	0.0668
		b ₂	0.0615
		b ₃	0.0586
B ₂	0.2358	b ₄	0.0641
		b ₅	0.0573
		b ₆	0.0597
		b ₇	0.0547
B ₃	0.2449	b ₈	0.0435
		b ₉	0.0725
		b ₁₀	0.0715
		b ₁₁	0.0574
B ₄	0.3324	b ₁₂	0.0656
		b ₁₃	0.0610
		b ₁₄	0.0623
		b ₁₅	0.0779
		b ₁₆	0.0656

Step 4: Calculate $W_i^{(1)}$ and $W_i^{(2)}$ of each database according to Equations (44) and (45). The results are shown in Tables 7 and 8.

Table 7. Weighted sum model $(W_i^{(1)})$.

Database	$W_i^{(1)}$
A ₁	$(S_{5.8095} e^{i2\pi S_{6.0309}}, S_{1.4804} e^{i2\pi S_{1.3722}}, S_{1.8560} e^{i2\pi S_{1.4716}})$
A ₂	$(S_{6.0503} e^{i2\pi S_{5.4583}}, S_{1.5185} e^{i2\pi S_{2.4230}}, S_{1.7671} e^{i2\pi S_{1.4686}})$
A ₃	$(S_{6.2033} e^{i2\pi S_{5.9038}}, S_{1.3419} e^{i2\pi S_{1.6033}}, S_{1.5043} e^{i2\pi S_{2.7089}})$
A ₄	$(S_{5.9655} e^{i2\pi S_{5.8074}}, S_{1.7989} e^{i2\pi S_{1.5232}}, S_{1.6171} e^{i2\pi S_{1.7702}})$

Table 8. Weighted product model $(W_i^{(2)})$.

Database	$W_i^{(2)}$
A ₁	$(S_{2.3609} e^{i2\pi S_{4.5130}}, S_{4.4523} e^{i2\pi S_{4.2112}}, S_{5.2866} e^{i2\pi S_{3.6801}})$
A ₂	$(S_{4.1802} e^{i2\pi S_{3.3065}}, S_{4.5621} e^{i2\pi S_{4.3459}}, S_{4.6478} e^{i2\pi S_{4.7373}})$
A ₃	$(S_{3.9462} e^{i2\pi S_{3.9033}}, S_{5.0627} e^{i2\pi S_{5.2598}}, S_{4.4704} e^{i2\pi S_{5.0270}})$
A ₄	$(S_{2.6061} e^{i2\pi S_{3.4999}}, S_{4.0515} e^{i2\pi S_{4.5264}}, S_{4.7324} e^{i2\pi S_{4.5300}})$

Step 5: Given $\beta = 0.9$, the assessment result combining the WASPAS method according to Equation (46) is W_i , as shown in Table 9.

Table 9. W_i values.

Database	W_i
A ₁	$(S_{5.4646} e^{i2\pi S_{5.8791}}, S_{1.7776} e^{i2\pi S_{1.6561}}, S_{2.1991} e^{i2\pi S_{1.6924}})$
A ₂	$(S_{5.8633} e^{i2\pi S_{5.2431}}, S_{1.8228} e^{i2\pi S_{2.6153}}, S_{2.0552} e^{i2\pi S_{1.7955}})$
A ₃	$(S_{5.9775} e^{i2\pi S_{5.7037}}, S_{1.7140} e^{i2\pi S_{1.9689}}, S_{1.8009} e^{i2\pi S_{2.9407}})$
A ₄	$(S_{5.6296} e^{i2\pi S_{5.5767}}, S_{2.0241} e^{i2\pi S_{1.8235}}, S_{1.9286} e^{i2\pi S_{2.0461}})$

Step 6: Calculate the score values of each database according to Equation (14).

$$S(\mathcal{F}_1) = 7.0048, S(\mathcal{F}_2) = 6.9716, S(\mathcal{F}_3) = 7.0335, S(\mathcal{F}_4) = 6.9795$$

Step 7: The ranking result of four databases is ranked for the quality assessment of emergency information.

$$A_3 > A_1 > A_4 > A_2$$

As a result of the above analysis, the quality of emergency information in database A₃ is the highest, the quality of emergency information in database A₁ is the second highest, database A₄ is in third place, and the worst quality of emergency information is in database A₂. Based on the ranking results, the main dimensions of the emergency information quality assessment index system to optimize the quality management of emergency information further to improve its quality can be the focus.

6.2. Sensitivity Analysis

There are four parameters involved in the calculation of the ALCT-SFDWPPHM operator, which are a , b , λ , and q . The values of the parameters are analyzed as follows.

- (1) When the parameters a , b take different values, the scores of each database will change accordingly. The score values and ranking of the four databases for emergency management information quality assessment are summarized in Table 10 (assuming $\lambda = 3, q = 3$).

From Table 10, it can be seen when a and b take the same value, the databases have the same score values, $S_1 = 6.9168, S_2 = 6.9149, S_3 = 6.9687, S_4 = 6.8759$, and the ranking result is $A_3 > A_1 > A_2 > A_4$. When $a = 5, b = 15$, the sorting result changes to $A_3 > A_1 > A_2 > A_4$, and A₄ becomes the worst database. When $a = 20, b = 10$, the ranking changes again to $A_3 > A_4 > A_1 > A_2$, with A₂ becoming the worst database.

Regardless of the values of a, b (a, b are not 0 simultaneously), the best database of each database evaluated with the ALCT-SFDWPPHM operator will remain unchanged and all of them will be A_3 .

Table 10. Score values and ranking of four databases when a, b change.

a, b	Score Values of Four Databases	Ranking
$a = 1, b = 1$	$S_1 = 6.9168; S_2 = 6.9149; S_3 = 6.9687; S_4 = 6.8759$	$A_3 > A_1 > A_2 > A_4$
$a = 1, b = 10$	$S_1 = 7.0048; S_2 = 6.9716; S_3 = 7.0335; S_4 = 6.9795$	$A_3 > A_1 > A_4 > A_2$
$a = 3, b = 7$	$S_1 = 6.9755; S_2 = 6.9559; S_3 = 7.0201; S_4 = 6.9636$	$A_3 > A_1 > A_4 > A_2$
$a = 5, b = 15$	$S_1 = 6.9257; S_2 = 6.9182; S_3 = 6.9714; S_4 = 6.8784$	$A_3 > A_1 > A_2 > A_4$
$a = 20, b = 10$	$S_1 = 6.9816; S_2 = 6.9717; S_3 = 7.0238; S_4 = 6.9955$	$A_3 > A_4 > A_1 > A_2$
$a = 10, b = 30$	$S_1 = 6.9721; S_2 = 6.9632; S_3 = 7.0268; S_4 = 6.9669$	$A_3 > A_1 > A_4 > A_2$
$a = 40, b = 50$	$S_1 = 6.9713; S_2 = 6.9545; S_3 = 7.0188; S_4 = 6.9652$	$A_3 > A_1 > A_4 > A_2$
$a = 50, b = 50$	$S_1 = 6.9168; S_2 = 6.9149; S_3 = 6.9687; S_4 = 6.8759$	$A_3 > A_1 > A_2 > A_4$

- (2) When the parameter λ is taken to a different value, the score values of each database will vary; at the same time, the sorting result will also be affected. The specific examples are illustrated in Figure 1 and Table 11 (assuming $a = 1, b = 10, q = 3$).

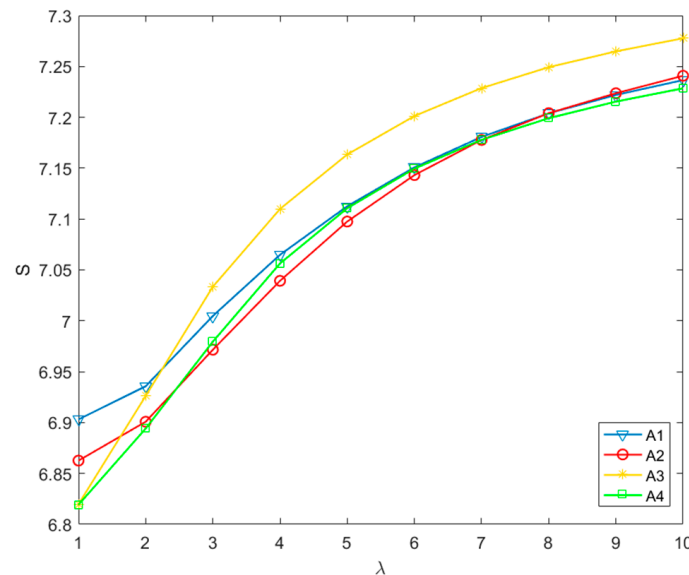


Figure 1. Score values of four databases when λ changes.

Table 11. Score values and ranking of four databases when λ changes.

λ	Score Values of Four Databases	Ranking
$\lambda = 1$	$S_1 = 6.9031; S_2 = 6.8626; S_3 = 6.8199; S_4 = 6.8193$	$A_1 > A_2 > A_3 > A_4$
$\lambda = 2$	$S_1 = 6.9357; S_2 = 6.9008; S_3 = 6.9262; S_4 = 6.8941$	$A_1 > A_3 > A_2 > A_4$
$\lambda = 3$	$S_1 = 7.0048; S_2 = 6.9716; S_3 = 7.0335; S_4 = 6.9795$	$A_3 > A_1 > A_4 > A_2$
$\lambda = 4$	$S_1 = 7.0649; S_2 = 7.0393; S_3 = 7.1100; S_4 = 7.0564$	$A_3 > A_1 > A_4 > A_2$
$\lambda = 5$	$S_1 = 7.1126; S_2 = 7.0976; S_3 = 7.1634; S_4 = 7.1106$	$A_3 > A_1 > A_4 > A_2$
$\lambda = 6$	$S_1 = 7.1508; S_2 = 7.1432; S_3 = 7.2012; S_4 = 7.1496$	$A_3 > A_1 > A_4 > A_2$
$\lambda = 7$	$S_1 = 7.1807; S_2 = 7.1777; S_3 = 7.2287; S_4 = 7.1779$	$A_3 > A_1 > A_4 > A_2$
$\lambda = 8$	$S_1 = 7.2039; S_2 = 7.2041; S_3 = 7.2492; S_4 = 7.1991$	$A_3 > A_2 > A_1 > A_4$
$\lambda = 9$	$S_1 = 7.2220; S_2 = 7.2236; S_3 = 7.2648; S_4 = 7.2154$	$A_3 > A_2 > A_1 > A_4$
$\lambda = 10$	$S_1 = 7.2365; S_2 = 7.2408; S_3 = 7.2777; S_4 = 7.2285$	$A_3 > A_2 > A_1 > A_4$

As can be seen from Figure 1 and Table 11, when $\lambda < 3$, the optimal database is A_1 and the worst database is A_4 . When $3 \leq \lambda \leq 7$, the sorting result is all $A_3 > A_1 > A_4 > A_2$, the best database is A_3 and the worst database is A_4 . When $\lambda > 7$, the ranking result is

$A_3 > A_2 > A_1 > A_4$, the best database is A_3 and the worst database is A_4 . The scores of databases A_1 , A_2 , A_3 , and A_4 all increase gradually with larger λ . It can be seen that the decision makers' preference can be reflected by the value of λ . If the decision makers prefer to choose the database A_1 , then the λ value less than 3 can be selected. Alternatively, they can choose the λ value greater than or equal to 3 if they tend to choose database A_3 .

- (3) The score values of each database will change accordingly when parameter q takes different values, thereby affecting the ranking results. For a specific example, please refer to Figure 2 and Table 12 (assuming $a = 1$, $b = 10$, $\lambda = 3$).

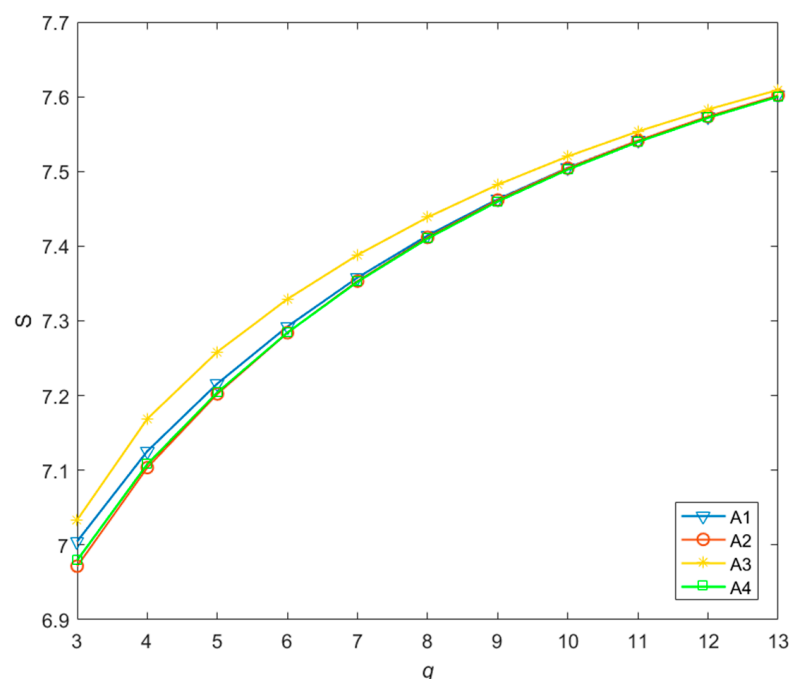


Figure 2. Score values of four databases when q changes.

Table 12. Score values and ranking of four databases when q changes.

q	Score Values of Four Databases	Ranking
$q = 4$	$S_1 = 7.1255; S_2 = 7.1035; S_3 = 7.1688; S_4 = 7.1081$	$A_3 > A_1 > A_4 > A_2$
$q = 5$	$S_1 = 7.2160; S_2 = 7.2022; S_3 = 7.2585; S_4 = 7.2041$	$A_3 > A_1 > A_4 > A_2$
$q = 6$	$S_1 = 7.2924; S_2 = 7.2841; S_3 = 7.3292; S_4 = 7.2843$	$A_3 > A_1 > A_4 > A_2$
$q = 7$	$S_1 = 7.3580; S_2 = 7.3532; S_3 = 7.3883; S_4 = 7.3524$	$A_3 > A_1 > A_2 > A_4$
$q = 8$	$S_1 = 7.4143; S_2 = 7.4118; S_3 = 7.4387; S_4 = 7.4103$	$A_3 > A_1 > A_2 > A_4$
$q = 9$	$S_1 = 7.4629; S_2 = 7.4617; S_3 = 7.4823; S_4 = 7.4599$	$A_3 > A_1 > A_2 > A_4$
$q = 10$	$S_1 = 7.5049; S_2 = 7.5045; S_3 = 7.5203; S_4 = 7.5025$	$A_3 > A_1 > A_2 > A_4$
$q = 11$	$S_1 = 7.5415; S_2 = 7.5414; S_3 = 7.5536; S_4 = 7.5396$	$A_3 > A_1 > A_2 > A_4$
$q = 12$	$S_1 = 7.5735; S_2 = 7.5736; S_3 = 7.5830; S_4 = 7.5719$	$A_3 > A_2 > A_1 > A_4$
$q = 13$	$S_1 = 7.6017; S_2 = 7.6019; S_3 = 7.6092; S_4 = 7.6003$	$A_3 > A_2 > A_1 > A_4$

From Figure 2 and Table 12, when $q < 7$, the sorting result is $A_3 > A_1 > A_4 > A_2$, the optimal database is A_3 and the worst database is A_2 . When $7 \leq q \leq 11$, the ranking result is $A_3 > A_1 > A_2 > A_4$. The best database is A_3 , while the worst database transitions from A_2 to A_4 . When $q > 11$, the ranking result is $A_3 > A_2 > A_1 > A_4$, the optimal database is A_3 and the second-ranked database is changed from A_1 to A_2 . The scores of databases A_1 , A_2 , A_3 , and A_4 all increase gradually with the increase of q . In addition, although the orders of A_1 , A_2 , A_3 , and A_4 change slightly when q takes a different value, whatever the any value of q , the optimal database is A_3 , which remains constant.

6.3. Qualitative Comparison

In general, a qualitative comparison can be made by comparing the characteristics of different methods. The proposed ALCT-SFDWPPHM operator is compared with CT-SFWA [65], CT-SFWG [65], CT-SFAAWA [66], CT-SFAAWG [66], CT-SDFWAA [67], CT-SDFWGA [67], CT-SFHWA [68], CT-SFHWG [68], LCT-SFPPWA, LCT-SFPPWG, LCT-SFDWPHM, and LCT-SFDWPGHM operators and the TOPSIS method. The characteristics of the comparison are: whether the parameter vector enhances the flexibility of the method, whether it considers the interrelationships between attributes, whether it takes into account the partitioning of the input parameters, and whether to reduce the negative effect. The specific analysis is shown in Table 13.

Table 13. Qualitative comparison of different methods.

Method	Whether the Parameter Vector Enhances the Flexibility of the Method	Whether It Considers the Interrelationships between Attributes	Whether It Takes into Account the Partitioning of the Input Parameters	Whether to Reduce the Negative Effect
CT-SFWA [65]	No	No	No	No
CT-SFWG [65]	No	No	No	No
CT-SFAAWA [66]	Yes	No	No	No
CT-SFAAWG [66]	Yes	No	No	No
CT-SDFWAA [67]	Yes	No	No	No
CT-SDFWGA [67]	Yes	No	No	No
CT-SFHWA [68]	Yes	No	No	No
CT-SFHWG [68]	Yes	No	No	No
CT-SFPPWA	No	No	Yes	Yes
CT-SFPPWG	No	No	Yes	Yes
LCT-SFDWPHM	Yes	Yes	Yes	No
LCT-SFDWPGHM	Yes	Yes	Yes	No
TOPSIS method	No	No	No	No
ALCT-SFDWPPHM	Yes	Yes	Yes	Yes

The proposed method incorporates the Dombi operations, which makes the aggregation operator more flexible by adjusting the parameter values. The application of the HM operator can better coordinate the relationships between attributes. Besides, the problem of uncorrelated multiple attributes in different partitions can be mitigated by taking into account the impact of input parameter partitioning. The proposed method has the ability to minimize the effect of singularities on the assessment results due to the inclusion of the PA operator. In addition, the advanced operator is aggregated, which effectively avoids the situation where the aggregated results are consistent or indistinguishable. In summary, the proposed method not only has the ideal flexibility in aggregating linguistic complex T-spherical fuzzy information and dealing with the interrelationships of the attributes, but also has the ability to minimize the negative impact of certain assessment value deviations.

6.4. Quantitative Comparison

In order to verify the feasibility and validity of the proposed multi-attribute assessment method, the ALCT-SFDWPPHM operator is compared with CT-SFWA [65], CT-SFWG [65], CT-SFAAWA [66], CT-SFAAWG [66], CT-SDFWAA [67], CT-SDFWGA [67], CT-SFHWA [68], CT-SFHWG [68], LCT-SFPPWA, LCT-SFPPWG, LCT-SFDWPHM, and LCT-SFDWPGHM operators and the TOPSIS method (assuming $a = 1$, $b = 10$, $\lambda = 3$, $q = 3$). The score values and ranking result of each database calculated by utilizing CT-SFWA [65], CT-SFWG [65], CT-SFAAWA [66], CT-SFAAWG [66], CT-SDFWAA [67], CT-SDFWGA [67], CT-SFHWA [68], CT-SFHWG [68], LCT-SFPPWA, LCT-SFPPWG, LCT-SFDWPHM, and LCT-SFDWPGHM operators, the TOPSIS method, and ALCT-SFDWPPHM operator are shown in Table 14.

Table 14. The score values and ranking result of different methods.

Method	Score Values of Four Databases	Ranking
CT-SFWA [65]	$S_1 = 0.0755; S_2 = 0.0633; S_3 = 0.0179; S_4 = 0.0321$	$A_1 > A_2 > A_4 > A_3$
CT-SFWG [65]	$S_1 = 0.2128; S_2 = 0.2523; S_3 = 0.2956; S_4 = 0.2625$	$A_3 > A_4 > A_2 > A_1$
CT-SFAAWA [66]	$S_1 = 0.7067; S_2 = 0.6943; S_3 = 0.7099; S_4 = 0.7041$	$A_3 > A_1 > A_4 > A_2$
CT-SFAAWG [66]	$S_1 = 0.1244; S_2 = 0.0509; S_3 = 0.1226; S_4 = 0.0897$	$A_1 > A_3 > A_4 > A_2$
CT-SDFWAA [67]	$S_1 = 0.7155; S_2 = 0.7023; S_3 = 0.7238; S_4 = 0.7188$	$A_3 > A_4 > A_1 > A_2$
CT-SDFWGA [67]	$S_1 = 0.3072; S_2 = 0.2782; S_3 = 0.3522; S_4 = 0.3106$	$A_3 > A_4 > A_1 > A_2$
CT-SFHWA [68]	$S_1 = 0.1377; S_2 = 0.1320; S_3 = 0.0986; S_4 = 0.1039$	$A_1 > A_2 > A_4 > A_3$
CT-SFHWG [68]	$S_1 = -0.2481; S_2 = -0.2842; S_3 = -0.3152; S_4 = -0.2872$	$A_1 > A_2 > A_4 > A_3$
CT-SFPPWA	$S_1 = 0.0693; S_2 = 0.0630; S_3 = 0.0253; S_4 = 0.0225$	$A_1 > A_2 > A_3 > A_4$
CT-SFPPWG	$S_1 = -0.2134; S_2 = -0.2401; S_3 = -0.2572; S_4 = -0.2621$	$A_1 > A_2 > A_3 > A_4$
LCT-SFDWPHM	$S_1 = 6.5474; S_2 = 6.5383; S_3 = 6.4796; S_4 = 6.5263$	$A_1 > A_2 > A_4 > A_3$
LCT-SFDWPGHM	$S_1 = 6.3642; S_2 = 6.3084; S_3 = 6.2450; S_4 = 6.2561$	$A_1 > A_2 > A_4 > A_3$
TOPSIS method	$S_1 = 0.5737; S_2 = 0.5065; S_3 = 0.3928; S_4 = 0.4443$	$A_1 > A_2 > A_4 > A_3$
ALCT-SFDWPPHM	$S_1 = 7.0048; S_2 = 6.9716; S_3 = 7.0335; S_4 = 6.9795$	$A_3 > A_1 > A_4 > A_2$

From Table 14, it can be found that the proposed ALCT-SFDWPPHM operator and the CT-SFAAWA [66] operator have the same ranking result, which is $A_3 > A_1 > A_4 > A_2$. The above comparative analysis strongly proves the applicability and effectiveness of the proposed operator.

- (1) Comparing with the complex T-spherical fuzzy weighted averaging (CT-SFWA) operator, the ranking result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, while the sorting result of the CT-SFWA [65] operator is $A_1 > A_2 > A_4 > A_3$. The best database obtained by utilizing the ALCT-SFDWPPHM operator is A_3 , while the worst database is A_3 by using the CT-SFWA operator. The reason is that the ALCT-SFDWPPHM operator aggregates the HM operator, which takes into account the correlation between attributes, and also aggregates the PA operator, which can reduce the negative impact of singular values. However, the CT-SFWA operator can only perform simple weighted aggregation and does not have the many superior properties of the ALCT-SFDWPPHM operator. Therefore, it can be concluded that the ALCT-SFDWPPHM operator proposed in this paper is more reliable than the CT-SFWA operator in terms of information assessment.
- (2) Comparing with the complex T-spherical fuzzy weighted geometric (CT-SFWG) operator, the ranking result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, while the ranking result of the CT-SFWG [65] operator is $A_3 > A_4 > A_2 > A_1$. Although the optimal database obtained by both the ALCT-SFDWPPHM and CT-SFWG operators is A_3 , the second, third, and fourth sorting results are different. Because the ALCT-SFDWPPHM operator aggregates the Dombi operations, which has more flexibility in information aggregation, and also aggregates the PHM operator that can fully consider the correlation of attributes in the same partition and the uncorrelation of attributes in the different partitions. However, the CT-SFWG operator can only accomplish the simple weighted aggregation process, without the unique function of the ALCT-SFDWPPHM operator. As a result, the conclusion can be drawn that the ALCT-SFDWPPHM operator has greater applicability and effectiveness than the CT-SFWG operator in evaluating information.
- (3) Comparison with the complex T-spherical fuzzy Aczel-Alsina weighted geometric (CT-SFAAWG) operator, the ordering result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, while the ordering result of the CT-SFAAWG [66] operator is $A_1 > A_3 > A_4 > A_2$. The optimal database is A_3 obtained by using the ALCT-SFDWPPHM operator, but the optimal database is A_1 obtained by using the CT-SFAAWG operator. The rationale behind this is that the ALCT-SFDWPPHM operator has the superior properties of the PA operator and PHM operator in information aggregation, which can eliminate the negative influence of extreme values on the assessment results and take into account

- the correlation between attributes. The CT-SFAAWG operator does not have any of the above characteristics. Therefore, it draws a conclusion that the ALCT-SFDWPPHM operator proposed in this paper has wider application than the CT-SFAAWG operator.
- (4) Compared with the complex T-spherical Dombi fuzzy weighted arithmetic averaging (CT-SDFWAA) operator and the complex T-spherical Dombi fuzzy weighted geometric averaging (CT-SDFWGA) operator, the sorting result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, whereas the ordering of both the CT-SDFWAA [67] operator and the CT-SDFWGA [67] operator is $A_3 > A_4 > A_1 > A_2$. The best database obtained by using the ALCT-SFDWPPHM operator, CT-SDFWAA operator and CT-SDFWGA operator is A_3 , and the worst database is A_2 , with only slight changes in the ranking results. The reason for this is that the ALCT-SFDWPPHM operator not only has great flexibility in information aggregation, but also can fully take into account the correlation between attributes, while the CT-SDFWAA operator and the CT-SDFWGA operator only have greater flexibility in the aggregation process, without paying attention to the correlation between attributes. Therefore, it can be concluded that the ALCT-SFDWPPHM operator is superior and more applicable than the CT-SDFWAA operator and CT-SDFWGA operator.
 - (5) Compared with the complex T-spherical fuzzy Hamacher weighted averaging (CT-SFHWA) operator and complex T-spherical fuzzy Hamacher weighted geometric (CT-SFHWG) operator, the sorting result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, but the ranking result of both the CT-SFHWA [68] and CT-SFHWG [68] operator is $A_1 > A_2 > A_4 > A_3$. The optimal database obtained by utilizing the ALCT-SFDWPPHM operator is A_3 , whereas the worst database given by using the CT-SFHWA and CT-SFHWG operator is A_3 . It shows that rankings in order have changed greatly. The primary explanation for this phenomenon is attributed to the fact that the ALCT-SFDWPPHM operator aggregates the HM operator that fully considers attribute correlations, and the PA operator that mitigates the negative effect of singular values on final assessment results during information aggregation, but the CT-SFHWA operator and the CT-SFHWG operator do not have these characteristics during the aggregation process. Therefore, the proposed ALCT-SFDWPPHM operator demonstrates that it is more scientific than the CT-SFHWA operator and CT-SFHWG operator in terms of assessment results.
 - (6) Compared with complex T-spherical fuzzy partitioned power weighted averaging (CT-SFPPWA) operator and complex T-spherical fuzzy partitioned power weighted geometric (CT-SFPPWG) operator, the ranking result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, whereas the ranking result of both the CT-SFPPWA and CT-SFPPWG operator is $A_1 > A_2 > A_3 > A_4$, respectively. The optimal database obtained through the utilization of the ALCT-SFDWPPHM operator is A_3 , whereas the least favorable database is A_2 . Conversely, employing both the CT-SFPPWA and CT-SFPPWG operator yields an optimal database of A_1 , with A_4 being deemed as the worst performing database. Consequently, there has been a significant alteration in the ranking situation. The reason is that the ALCT-SFDWPPHM operator not only reduces the influence of extreme values on the final assessment results, but also has great flexibility in information aggregation. The CT-SFPPWA and CT-SFPPWG operator do not have the flexibility property of the Dombi operations during the aggregation process. Therefore, it can be seen that the ALCT-SFDWPPHM operator proposed in this paper is more effective than the CT-SFPPWA operator and CT-SFPPWG operator in information assessment.
 - (7) Compared with linguistic complex T-spherical fuzzy Dombi weighted partitioned Heronian mean (LCT-SFDWPHM) operator and linguistic complex T-spherical fuzzy Dombi weighted partitioned geometric Heronian mean (LCT-SFDWPGHM) operator, the ranking in order of the proposed operator is $A_3 > A_1 > A_4 > A_2$, while the sorting result of both the LCT-SFDWPHM operator and LCT-SFDWPGHM operator is $A_1 > A_2 > A_4 > A_3$. The optimal database obtained through using the

ALCT-SFDWPPHM operator is A_3 , while the worst database obtained using the LCT-SFDWPHM and LCT-SFDWPGHM operator is both A_3 . A huge change can be seen in the sorting results, because the ALCT-SFDWPPHM operator combines the PHM operator and PA operator, enabling consideration of attribute correlations during information aggregation and minimizing the impact of singular values on final assessment results. However, both the LCT-SFDWPHA operator and LCT-SFDWPGHM operator only consider attribute correlations during the aggregation process without accounting for negative effects caused by singular values. Therefore, it can be inferred that the proposed ALCT-SFDWPPHM operator exhibits a broader scope of application in information assessment compared to both the LCT-SFDWPHM operator and LCT-SFDWPGHM operator.

- (8) Compared with the linguistic complex T-spherical fuzzy TOPSIS method, the sorting result of the proposed operator is $A_3 > A_1 > A_4 > A_2$, while the sorting result using the TOPSIS method is $A_1 > A_2 > A_4 > A_3$. The optimal database obtained by using the ALCT-SFDWPPHM is A_3 and the worst database is A_2 , but the optimal database is A_1 and the worst database is A_3 using the TOPSIS method. The sorting situation appears to be significantly different. The reason for this is that the ALCT-SFDWPPHM operator not only has great flexibility in information aggregation, but also can take into account the correlations between the attributes of the inter-subdivision area and can reduce the influence of the singular values on the final assessment results. However, the TOPSIS method does not have the functional properties of the Dombi operations, PA, PHM and advanced operators during the aggregation process. Therefore, it can be concluded that the proposed ALCT-SFDWPPHM operator is more applicable and superior to the TOPSIS method in terms of information assessment.

Through quantitative comparative analysis, it is proven that the proposed ALCT-SFDWPPHM operator has great flexibility in the process of information aggregation, which can fully take into account the correlation between attributes in the partition and the uncorrelation between attributes in different partitions, so as to reduce the distortion in the process of aggregation. At the same time, it can avoid the impact of the irrational judgments made by decision makers due to the limitation of time or experience on the overall assessment results. In conclusion, the proposed multi-attribute assessment method has greater flexibility, stronger applicability, and wider application range.

6.5. Discussion

In Section 6.2, the sensitivity analysis discusses the effects of the changes of four parameters on the final assessment results, as shown in Tables 10–12. Section 6.3 is the comparative analysis, and the ALCT-SFDWPPHM operator proposed in this paper is compared with 13 aggregation methods from qualitative and quantitative perspectives, highlighting the superiority and effectiveness of the operator proposed in this paper, as shown in Tables 13 and 14. Next, this paper will be discussed in the following perspectives: (1) technical contribution of the proposed novel operator for uncertain information fusion; (2) specific advantages of the proposed fuzzy multi-attribute assessment method; (3) implications of the results analysis for emergency information quality management practices.

In this paper, we propose LCT-SFS, which is used to characterize the assessment value of each attribute and has more expressive advantages over existing fuzzy sets, such as LTS [69], which can only express qualitative uncertainty information at different levels of granularity. T-SFS [70] can only express quantitative uncertainty information containing membership, abstinence, and non-membership degrees. CT-SFS [71] can express only two-dimensional quantitative uncertainty information containing the degrees of membership, abstinence, and non-membership. LCT-SFS combines the information expression advantages of the above fuzzy sets and expands the expression range, which can describe the two-dimensional fuzzy assessment information of qualitative-quantitative fusion more flexibly and comprehensively. Taking LCT-SFS as the object of analysis, the ALCT-SFDWPPHM operator, which has more comprehensive advantages than the independent combination of

various types of fuzzy sets with Dombi [72], PA [73], and PHM [74] operators, is proposed. The operator can simultaneously deal with the problem of multi-dimensional attribute partitioning and the existence of correlation of attributes in each zone, reduce the influence of singularities in attribute values on the perturbation of the calculation results and improve the resolution of the aggregation results of the existing basic operator, which is a strong and favorable tool for solving the aggregation of uncertain information.

In this paper, we propose a multi-attribute assessment method based on the ALCT-SFDWPPHM operator using the linguistic complex T-spherical fuzzy entropy measure to obtain the attribute weights. We apply the ALCT-SFDWPPHM operator to aggregate the attribute assessment information and the WASPAS method for the final assessment object sorting. From the perspective of user's cognition and emotional experience, an index system of the quality assessment of emergency information is constructed. It is a theoretical and methodological expansion of the existing research work and is conducive to promoting the quality of emergency information and enhancing the efficiency of emergency information management. Compared with the existing emergency information quality assessment methods, such as simple weighted average [75], TOPSIS [9], etc., which do not consider the inherent multi-dimensionality of the assessment index system that requires partition aggregation nor the strong correlation between indices under the same dimension, the method proposed in this paper solves the above problems and has high adaptability.

We ranked the quality of emergency information in the four databases through calculation and analysis. For the databases with the worst ranking of the quality of emergency information, we can start from the index system to check and fill in the gaps at the micro level. After objectively assigning weights, the usefulness of information was determined to be the most important of the indices, which is in line with the existing research. The usefulness of emergency information directly determines the effectiveness of handling emergencies [76]. With emergencies, the quality of information varies, and "information fog" has become a problem that cannot be ignored. Therefore, the government should enhance the ease of use of the search function and the accuracy of the search results to improve the usefulness of emergency information because it is the most important factor in improving the quality of emergency information, which can provide strong support for emergency management. The next most important indices are simplicity and standardization of information, which have the second and third highest weights among all indices. These findings are in line with the findings of Wong et al. [77], who concluded that better results are achieved when information is distributed in a way that it is sent in multiple messages, brief and formal. Therefore, the government should improve relevant policies and regulations, formulate detailed rules on the quality requirements of emergency information, and quickly present emergency information with simplicity and standardization so that managers can quickly understand emergency information and make corresponding emergency decisions. The index of reliability of information is also a key factor in determining the quality of information, and its importance is located in the fourth place among the indices. Our view is equally supported by the findings of Agrawal et al. [78], who concluded that the source reliability index occupies an indispensable place in assessing the quality of information. Therefore, the government should clarify the responsibilities and obligations of emergency information management and provide legal safeguards for the reliability of emergency information through legislative means to enhance the credibility of the government's emergency information management and ensure the authority and reliability of emergency information. The government can conduct detailed research on the index system and propose targeted and in-depth improvement policies based on the findings. We will not elaborate on every one of them here due to the space limitations of this paper. However, it is believed that implementing these policies will enable the government to gradually optimize the quality management of emergency information and provide solid information security for emergency management.

7. Conclusions

The core contribution of this paper is that it proposes a new fuzzy multi-attribute assessment methodology and applies it to solve the real emergency information quality assessment problem. First, we define a new kind of fuzzy set, the LCT-SFS, and give its basic operations and information measures. The LCT-SFS is more widely applicable in expressing uncertain information, which enriches the theoretical scope of the traditional fuzzy set. On this basis, the ALCT-SFDWPPHM operator is proposed, which has the following advantages in uncertain information aggregation: (1) the operator takes the more compatible Dombi operations as the underlying rule and applies it to the computational process of the aggregation operator, making it more general and flexible in the process of information aggregation; (2) this operator combines the advantages of the PA operator and can effectively reduce the negative impact of singularities in the assessment information on the assessment results; (3) this operator integrates the advantages of the PHM operator, which can solve the problem of multi-dimensional assessment information aggregation through partitioning and consider the correlation between attributes under each dimension; (4) this operator joins the advantages of the advanced operator, which can improve the resolution of the calculation results and circumvent the indistinguishable situation of the aggregation results. Furthermore, a multi-attribute assessment method based on the ALCT-SFDWPPHM operator and the WASPAS method is constructed, which can scientifically deal with the complex and uncertain multi-dimensional and multi-attribute assessment information aggregation problem. It has a greater advantage in assessing complex systems with multi-structures, multi-types, multi-objectives, and multi-attributes. Then, to address the problem of the quality assessment of emergency information, a hierarchical structure model of emergency information quality assessment indices was constructed from the perspective of the user's cognition and emotional experience. Finally, the applicability and superiority of the method proposed in this paper are verified through the arithmetic example analysis of the emergency information quality assessment problem, which can effectively guide the work of the quality assessment of emergency information and improve emergency information quality.

Although the research in this paper has a certain superiority over existing research results, enriches the theoretical scope of fuzzy sets, and expands the application boundary of fuzzy multi-attribute assessment methods, it still has certain limitations. First, the LCT-SFS proposed in this paper is rooted in the traditional fuzzy set and has not yet been considered for fusion research with rough set and soft set, which also have greater advantages for representing uncertain information [79–82]. In future research, efforts will be made to conduct interdisciplinary studies of fuzzy sets, rough sets, and soft sets to contribute to uncertain multi-attribute decision theory sustainably. Second, the proposed operators in this paper can only solve the correlation problem between two attributes but have been unable to solve the correlation problem among multiple attributes. In future research, the LCT-SFS proposed in this paper can be combined with the Muirhead mean operator, which is capable of dealing with correlations among multiple attributes to apply them to a wider range of research areas, for example, in the field of information sharing assessment, which includes research on information resource sharing between government organizations, between formation agencies of journal literature, or between judicial and law enforcement departments. It can also be applied to the information response capacity assessment, in which the emergency response capacity of urban communities after emergencies and the assessment of the emergency response capacity of suppliers to industrial supply chains also have a large scope of application.

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Nomenclature

List of Symbols

F	A complex T-spherical fuzzy set	$\tilde{\gamma}_F(x)$	The membership degree of a complex T-spherical fuzzy number
$\tilde{\delta}_F(x)$	The abstinence degree of a complex T-spherical fuzzy number	$\tilde{\tau}_F(x)$	The non-membership degree of a complex T-spherical fuzzy number
$\varsigma_F(x)$	The hesitant degree of a complex T-spherical fuzzy number	\mathcal{F}	A complex T-spherical fuzzy number
S	A linguistic term set	$B_{D, \lambda}(h, l)$	The Dombi t-norm
$B_{D, \lambda}^c(h, l)$	The Dombi t-conorm	$Sup(x_i, x_j)$	The support of x_j for x_i
$T(K_i)$	The sum of $Sup(K_i, K_j)$	K	A linguistic complex T-spherical fuzzy number
$\tilde{S}_\gamma(k)$	The linguistic membership degree of a linguistic complex T-spherical fuzzy set	$\tilde{S}_\delta(k)$	The language abstinence degree of a linguistic complex T-spherical fuzzy set
$\tilde{S}_\tau(k)$	The linguistic non-membership degree of a linguistic complex T-spherical fuzzy set	$S_\varsigma(k)$	The refusal degree of a linguistic complex T-spherical fuzzy set
$S(K)$	The score function of linguistic complex T-spherical fuzzy number K	$A(K)$	The accuracy function of linguistic complex T-spherical fuzzy number K
$d(K_1, K_2)$	The Hamming distance between K_1 and K_2	K^+	The maximum value of a linguistic complex T-spherical fuzzy number K
K^-	The minimum value of a linguistic complex T-spherical fuzzy number K	$\theta(\mathcal{K})$	$\left(\frac{\mathcal{K}^q}{t^q - \mathcal{K}^q}\right)^\lambda$
$\eta(\mathcal{K})$	$\left(\frac{t^q - \mathcal{K}^q}{\mathcal{K}^q}\right)^\lambda$	w'_i	$\frac{1+T(K_{fi})}{\sum_{i=1}^n (1+T(K_g))}$
$\mathcal{J}(h_{fi})$	$\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fi})^q}{t^q - (\rho h_{fi})^q}\right)^\lambda$	$\mathcal{J}(h_{fj})$	$\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{(\rho h_{fj})^q}{t^q - (\rho h_{fj})^q}\right)^\lambda$
$\mathcal{H}(h_{fi})$	$\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fi})^q}{(\rho h_{fi})^q}\right)^\lambda$	$\mathcal{H}(h_{fj})$	$\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fj})^q}{(\rho h_{fj})^q}\right)^\lambda$
$\aleph(u)$	$\left(\frac{(\rho u)^q}{t^q - (\rho u)^q}\right)^\lambda$	$\mathcal{H}(u)$	$\left(\frac{t^q - (\rho u)^q}{(\rho u)^q}\right)^\lambda$
$\mathfrak{J}(h_{fi'})$	$\frac{nw'_{fi}w_{fi}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fi'})^q}{(\rho h_{fi'})^q}\right)^\lambda$	$\mathfrak{J}(h_{fj'})$	$\frac{nw'_{fj}w_{fj}}{\sum_{g=1}^n w_g w'_g} \left(\frac{t^q - (\rho h_{fj'})^q}{(\rho h_{fj'})^q}\right)^\lambda$

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