

Article

Construction of a Dynamic Diagnostic Approach for a Fuzzy-Interval Petri Network

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Abstract: Fault diagnosis plays a crucial role in enhancing system dependability and minimizing potential catastrophic consequences for both equipment and human safety. This article presents a research study focused on developing a diagnosis and control approach for discrete event systems using the Petri net Fuzzy Interval (IFPN). The Petri net is utilized as a modeling tool for the target system. The paper describes a case study conducted on an ingredient mixing system, where the objective is to maintain the concentration of ingredients within a valid range. A diagnostic framework is constructed and successfully applied to identify faults in the system. The proposed approach is further validated through simulation tests conducted on a mixing system.

Keywords: uncertain systems; intervals fuzzy Petri nets; modeling; fault diagnosis



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1. Introduction

In the literature, most applications of fuzzy theory to automatic control systems are basically directed toward the development of mathematical models or fuzzy logic controllers for linear and nonlinear systems with given or unknown system models. However, few applications on real systems are presented [1].

This paper presents an approach for the modeling and design of an automatic control real system in a fuzzy environment, where we consider that the input of the system is variable and the finished product depends on the variability in the value of the raw material at the input of the system.

Our goal is the construction of a robust control to ensure product quality parameters at the output. In addition, the constraint to be guaranteed does not depend on time. In terms of modeling, the introduction of a new tool is useful. In fact, the control of a process under interval constraints is for any quality [2]. So, we flatten out a model that integrates the uncertain aspect in the form of fuzzy intervals and use the built model for defect diagnosis. The simulation results of our approach prove its validity in the process under consideration [3].

This work is devoted to the construction of a diagnostic system capable of detecting defects in an uncertain system. For this, we use our developed model, which describes the mixing process.

First, we present the interval theory, specifically the fuzzy interval and Fuzzy Petri Net Model [4]. Second, we propose a new modeling approach for the detection of defects in uncertain systems. This modeling approach combines two tools, namely the MOP and the interval approach. A statistical approach is presented to construct the ranges of validity of the fuzzy intervals assigned to the places of the constructed MOP model [1].

Our approach is validated by a simulation program to prove its validity and robustness.

2. Fuzzy Approach

The introduction of fuzzy set and interval theory is increasingly being applied in the field of modeling and analysis of uncertain systems. Unlike classical set theory, where intervals describe only all possible values and from a membership point of view, it is all or nothing: an element belongs or does not belong to a set. Fuzzy Subset Theory (FSS) is more suited to represent all qualitative knowledge, especially when it comes to manipulating data and exploiting vague or imprecise data. This theory gave rise to the notion of fuzzy intervals that take into account, in their definition, the degree of uncertainty for possible values [5]. For an element of information, the theory of fuzzy subsets gives the possibility of belonging to a set, at a level of membership ranging from 0 to 1.

2.1. Fuzzy Subset Concept (SEF)

The fuzzy subset concept was introduced by L.A. It is based on a degree of belonging, which generalizes characteristic functions and thus allows for modeling the human representation of knowledge and improving the performance of the decision systems using modeling [5]. This is because, in classical theory, this notion of belonging is rigid; for an X set, an element belongs or not to a subset of X . On the other hand, in the fuzzy approach, if one considers a reference set X [6], a fuzzy subset A of this reference is characterized by a function of belonging A ; fuzzy subset A of a set X is a function $A: X \rightarrow L$, where L is the interval $[0, 1]$. This function is also called a membership function. A membership function is a generalization of an indicator function (also called a characteristic function) of a subset defined for $L = \{0, 1\}$. More generally, one can use any complete lattice L in a definition of a fuzzy subset A (Figure 1).

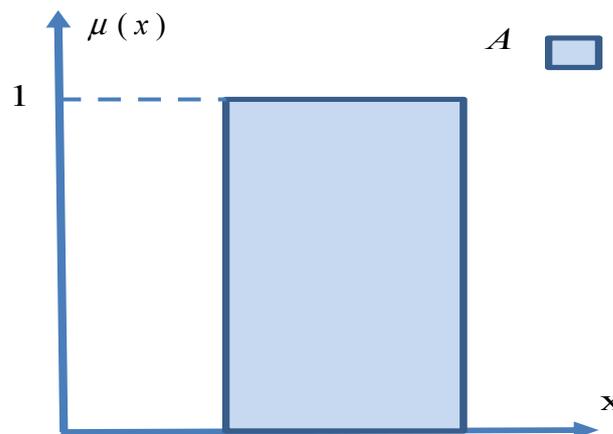


Figure 1. Example of a subset characteristic function.

For example, consider the following set $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We can define the fuzzy subset of numbers approximately equal to 5 by

$$A = 5 = \{(0, 0), (1, 0), (2, 0), (3, 0), (4, 0.5), (5, 1), (6, 0.5), (7, 0), (8, 0), (9, 0), (10, 0)\}$$

An instant subset may be represented as a triangular, trapezoidal, or parabolic function. In the case of fuzzy subsets [5], the membership function can be defined in trapezoidal form, as shown in Figure 2.

The purpose of the concept of SEF is to allow gradations in the membership of an element X to class A , that is, to allow an element to belong strongly to that class. This notion of belonging is generally represented in the form of a function. Figure 2 shows three concepts used in the theory of fuzzy subsets [7],

- The support of the fuzzy subset is the set of elements x of X such as $\mu_{A(x)} \geq 0$;
- The core of the fuzzy subset is the set of elements x of X such as $\mu_{A(x)} = 1$;
- The support of the fuzzy subset is the set of elements x of X such as $\mu_{A(x)} \geq 0$;

- The height h indicates the degree of belonging of element X to the set.
An element X is either in A or excluded; it can never partially belong to the set.

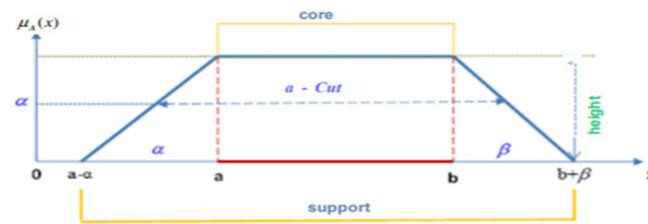


Figure 2. Trapezoidal membership function for a fuzzy subset.

2.2. Fuzzy Intervals

Definition 1 A fuzzy subset A of X is said to be normal if $\mu_A(x) = 1$. It is said to be convex if

$$\forall x, y \in X, \forall \lambda \in [0, 1] : \mu_A(\lambda x + (1 - \lambda)y) \geq \min(\mu_A(x), \mu_A(y)) \tag{1}$$

A fuzzy interval represents an interval whose edges are poorly defined of the type ‘between a and b ’ [7]. A fuzzy number is used to represent a fuzzy assessment of the ‘‘about’’ type. A singleton allows entry into a system that manipulates only symbolic quantities (Figure 3).

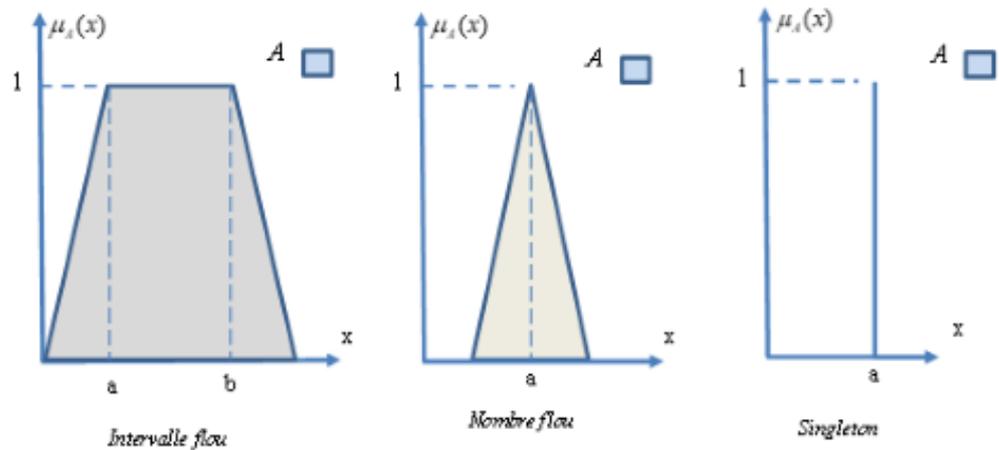


Figure 3. Examples of particular insane subsets.

The interest of our choice of fuzzy interval lies in its capacity for the correct representation of imprecise quantities. This is the case with our study on modeling for the diagnosis of uncertain systems. In this context, fuzzy numbers and singletons can be used for a unified representation of symptoms [6].

2.3. Fuzzy Interval Operations

In the following, we are interested in the fuzzy operations applied to the types of fuzzy quantities of particular shapes leading to the simplification of the calculations: the membership functions of the trapezoidal and the triangular shape [8]. Let us consider Figure 2 and the fuzzy interval $(a - \alpha, b + \beta)$; the linguistic label characterizes the support of this function where $(a - \alpha)$ and $(b + \beta)$ are, respectively, the lower and upper limits of the fuzzy interval. These can be represented from a quadruplet (a, α, b, β) , with $a < b$, $[a, b]$ as the modal value of the interval [9].

The expression of the corresponding trapezoidal membership function in Figure 2 is written as follows:

$$\mu_A(x) = \begin{cases} \left\{ \frac{x-a+\alpha}{\alpha}, \text{ pour } x \in [a-\alpha, a] \right. \\ 1, \text{ pour } x \in [a, b] \\ \left. \left\{ \frac{b+\beta-x}{\beta} \text{ pour } x \in [b, b+\beta] \right. \right. \\ 0, \text{ ailleurs} \end{cases} \quad (2)$$

when $a = b$, there will be a triangular membership function (Figure 4).

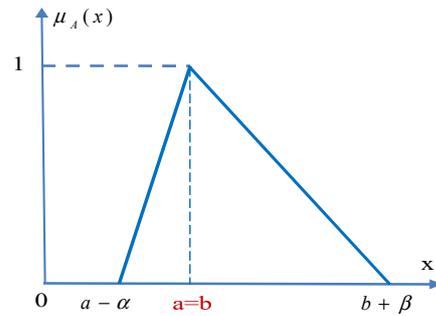


Figure 4. Triangular membership function.

The arithmetic operations on the intervals can be generalized, and for two given fuzzy intervals A and B defined $A = [A^-, A^+]$ $B = [B^-, B^+]$, we write

$$A + B = [A^- + B^-, A^+ + B^+] \quad (3)$$

$$A - B = [A^- - B^+, A^+ - B^-] \quad (4)$$

$$A \times B = [\min(A^- \times B^+, A^+ \times B^-, A^- \times B^+, A^+ \times B^-), \max(A^- \times B^-, A^+ \times B^-, A^- \times B^+, A^+ \times B^+)] \quad (5)$$

Example: Let us consider the intervals of the functions of Figure 5. If we apply the rule of addition to functions h1 and h2, we obtain function h3 [10].

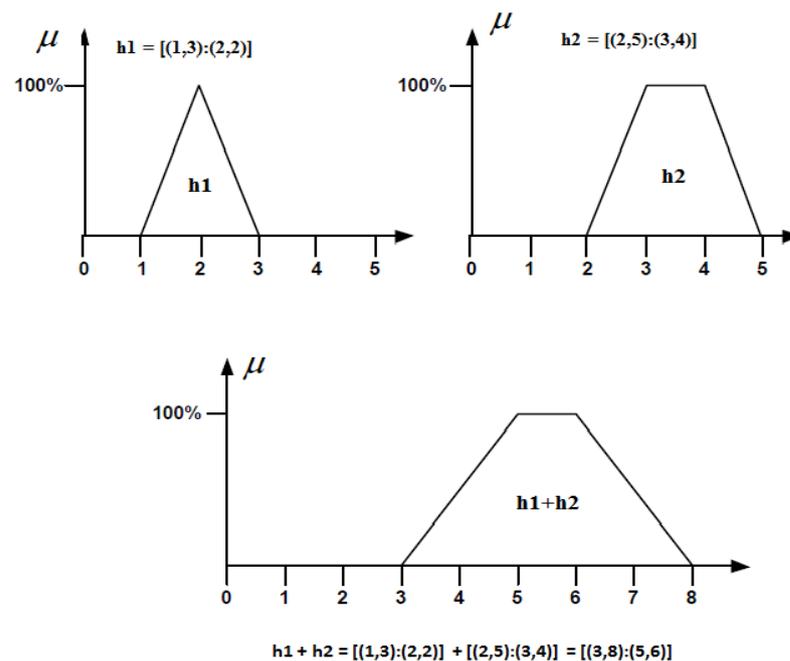


Figure 5. Example of blurry interval operations.

In this study, we combine two approaches: In the first, we establish the interval bounds using mathematical equations, and in the second, we detect faults using rules. To accomplish this, we must first create a kind of arithmetic system that permits operations on qualitative numbers. Addition and subtraction are the only necessary fundamental operations [11].

2.4. Qualitative Rules and Fuzzy Relationships

The representation of qualitative knowledge relevant to a given domain can be achieved using the IF-THEN fuzzy logic concept, which is frequently used to describe the following form of logical reliance between variables:

If V_1 is K_1 and V_n is K_n , then U is M , with $K_1 \dots, K_n$, and M acting as predicates to describe $V_1 \dots, V_n$, and U [8].

In our work, we are interested in diagnosing defects. So, within this framework, we are going to have two fuzzy propositions: one about symptoms and the other about defects.

In the field of diagnosis, the rules are written as follows:

$$R_k : \text{If } P_k(x) \text{ Then } P_k(y), k = 1 \dots n \tag{6}$$

where

n : the number of rules;

$P(x)$ and $P(y)$: Fuzzy proposals on observable symptoms and defects, respectively.

The integration of qualitative rules and fuzzy relationships is performed in MOPs [8].

In our work, we introduce a tool for modeling uncertain systems. This tool integrates fuzzy logic into a Petri model. This modeling method provides a framework for exploiting fuzzy knowledge that qualitatively yields more refined results than conventional logic. The tool thus constructed will be adapted in the modeling of detection/decision functions by a place approach at fuzzy intervals [12].

Consider the following linguistic description:

Rule 1: U is M_1 if V_1 is X_{11} AND \dots AND V_n is $X_{1n} \dots$

If V_1 is X_{M1} AND \dots AND V_n is X_{mn} , then U is M_m according to rule m [2].

It is clear how fuzzy PN and the linguistic phrase “IF THEN rules” are related. The relationship between the Petri network and the “IF—THEN” rules is depicted in Figure 6.

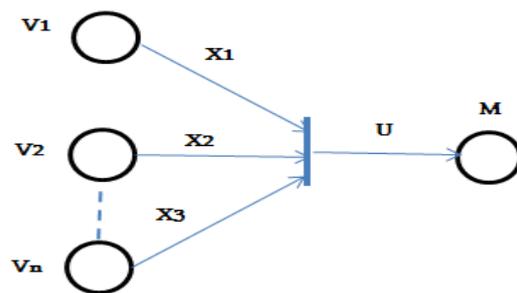


Figure 6. Linguistic description matches an IF-THEN rule.

2.5. Fuzzy Rule, RdP, and Fault Tree Mapping

In order to model the detection function, this RdP modeling tool is proposed, which integrates the temporal aspect—the instant appearance of several defects in the monitored system. This model is targeted for fuzzy logic rule modeling that results from the fault tree logic expression, which is identified at the monitored system level.

We are interested in failure trees constructed only of “AND” and “OR” logic gates and corresponding logic operations [13],

The fault tree uses the most unexpected fault event as its scan target and goes through each potential contributing element one by one. After choosing the appropriate symbols

to represent the events, we connect them to the top-level events, intermediate events, and fundamental events to create the graph (Figure 7).

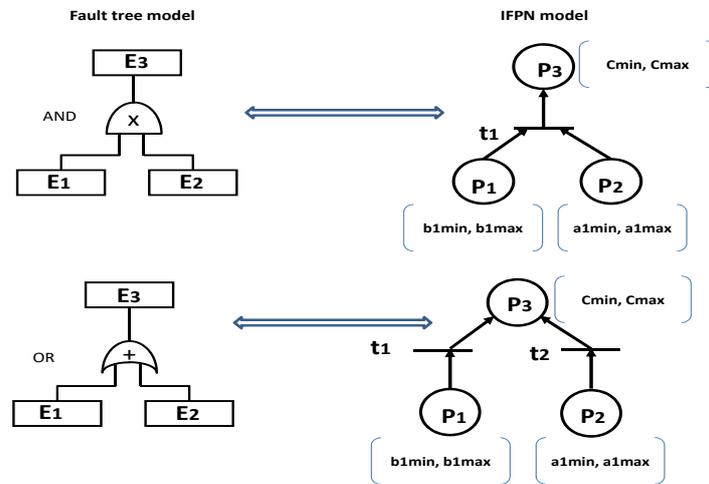


Figure 7. The defect tree’s logical circuits and the accompanying model Petri nets.

3. Illustration of the Modeling Approach

3.1. Description of the Process to Be Modeled

The mixing process in Figure 8 will serve as an illustration of our strategy for modeling uncertain systems for fault identification [13]. In order to create Ajax water for glass panes, two separate ingredients are combined with water in this method. Its concentration affects the final product’s quality. When deciding whether or not to approve the product, this index is taken into account [14].

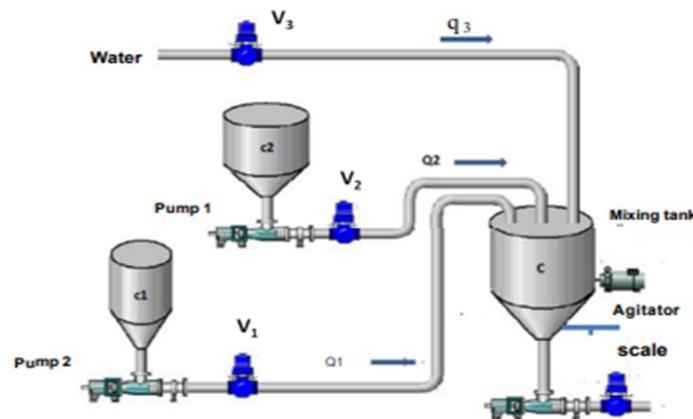


Figure 8. Mixing System.

Through the two valves V1 and V2, two ingredients Q1 and Q2, respectively, in tanks R1 and R2 are discharged into the mixing tank. To create Ajax water for windows, a specified amount of water will be added to this mixture and swirled for a predetermined amount of time using a stirrer. The appropriate valves are used to change the amount of each ingredient as well as the amount of water. The entire amount must not be more than 95% of the tank’s overall capacity. The batch will automatically be weighed following the shaking process. The product is then packaged in bottles for marketing. The value of the concentration, which we wish to keep constant, affects the product’s quality.

3.2. Model Construction

The concentration of the resultant mixture is largely related to the stresses that are examined. The model that will be created must be able to explain how this concentration’s

value changes over time. We believe that this variation is a result of changes in the relative concentrations of the two products to be combined as well as changes in the amount of water used in the mixture [15].

The concentration value must be kept within a predetermined range by our management. The latter should be viewed as the fuzzy interval's center. The intermittent use of the Petri network theorems and properties [16] is anticipated to be the same as that of our Blurred RoP model. A first-order differential around a reference point is utilized to quantify the fluctuation in concentration at the different stations of the process, allowing us to linearize the model. Because one is still very close to an equilibrium point, this linearization property is employed to move within the space of valid solutions corresponding to each interval's limit.

Fast systems make it impossible to control every manufacturing unit. After sampling, the worth of an average is assessed.

It is a batch and recurring operation in our situation. Managing the variations in product quality upon entry, which can vary depending on each ingredient, is carried out along with controlling the variations in concentration on each lot.

The first ingredient's concentration C1, the second ingredient's concentration C2, and the amount of water added, together represented as C, determine the mixture's concentration. Adjustment operations carried out at the supply valves to control the flow rates can be used to ensure the correction of variations in the mixture's concentration.

An adjustment can be made to control the new value(s) of C1 and C2 while simultaneously affecting the volume of water to be added.

3.3. Fuzzy Interval Petri Grating Modeling

In order to solve the issue of concentration variation and thus the range constraints on this concentration [17], we must therefore take into account the other three parameters. The following equation is considered to give the relationship between the C concentration and the ingredient masses (for each lot produced):

$$C = \frac{m_1 + m_2}{V_e} \tag{7}$$

where C is the concentration of the final product in kg/mm³;

m₁ is the mass of ingredient 1 in kg;

m₂ is the mass of ingredient 2 in kg;

V_e volume of water in mm³.

m₁ = C1 * V1 and M2 = C2 * V2.

where Ci and Vi are the product Qi (i = 1, 2) concentrations (in kg/mm³) and volumes (in mm³), respectively.

This connection can be expressed as follows:

$$C = \frac{C_1 h_1 S + C_2 h_2 S}{V_e} \tag{8}$$

The following are the parameters' interval restrictions:

C [Cmin, Cmax: The product will be rejected if the concentration C value falls outside this range.

C1 [C1 min, C1 max].

C2 [C2 min, C2 max].

V_e represents the amount of water in mm³: V ∈ [V_e min, V_e max].

S is the tank section measured in mm².

$$C_{\min} \leq \frac{C_1 h_1 S + C_2 h_2 S}{V_e} \leq C_{\max} \tag{9}$$

The tolerances for each parameter must be determined before building the model. To do this, we suggest a statistical approach based on precise values obtained from statistical production data. The procedure for using this method of computation and the presumptions taken into account are described in the next subsection.

3.4. Linear Approximation

We suppose that there is very little variance in the parameters around the mean. The relationship between the order near the reference establishment and the linearization approach can therefore be approximated as follows:

$$\Delta C \in [\Delta C_{min}, \Delta C_{max}]$$

Let $C_0, C_{10}, h_{10}, C_{20}, h_{20}$, and Ve_0 be the desired values for parameters C, C_1, h_1, C_2 , and Ve , respectively. After linearizing the equation around the operating point [17], we can write

$$C \approx C_0 + \frac{\delta C}{\delta c_1}(c_1 - c_{10}) + \frac{\delta C}{\delta h_1}(h_1 - h_{10}) + \frac{\delta C}{\delta c_2}(c_2 - c_{20}) + \frac{\delta C}{\delta h_2}(h_2 - h_{20}) + \frac{\delta C}{\delta Ve}(Ve - Ve_0) + \frac{1}{2} \frac{\delta^2 C}{\delta c_1^2}(c_1 - c_{10})^2 + \frac{1}{2} \frac{\delta^2 C}{\delta h_1^2}(h_1 - h_{10})^2 + \frac{\delta^2 C}{\delta c_2^2}(c_2 - c_{20}) + \frac{1}{2} \frac{\delta^2 C}{\delta h_2^2}(h_2 - h_{20})^2 + \frac{1}{2} \frac{\delta^2 C}{\delta Ve^2}(Ve - Ve_0)^2 \tag{10}$$

which can be in the form of

$$C \approx C_0 + \sum_{i=1}^5 [b_i X_i + b_{ii} X_i^2] \tag{11}$$

with

$$X_1 = \frac{(c_1 - c_0)}{\sigma_1} \quad X_2 = \frac{(h_1 - h_0)}{\sigma_2} \quad X_3 = \frac{(c_2 - c_{20})}{\sigma_3} \quad X_4 = \frac{(h_2 - h_{20})}{\sigma_4} \\ X_5 = \frac{(Ve - Ve_0)}{\sigma_5}$$

and

$$b_i = c_i \sigma_i \quad b_{ii} = c_{ii} \sigma_i^2 \tag{12}$$

where

$$b_1 = \frac{\delta C}{\delta c_1} \quad b_2 = \frac{\delta C}{\delta h_1} \quad b_3 = \frac{\delta C}{\delta c_2} \quad b_4 = \frac{\delta C}{\delta h_2} \quad b_5 = \frac{\delta Ve}{\delta Ve} \quad b_{11} = \frac{\delta^2 C}{\delta c_1^2} \quad b_{22} = \frac{\delta^2 C}{\delta h_1^2} \quad b_{33} = \frac{\delta^2 C}{\delta c_2^2} \\ b_{44} = \frac{\delta^2 C}{\delta h_2^2} \quad b_{55} = \frac{\delta^2 C}{\delta Ve^2} \tag{13}$$

b is the parameter where standard deviation is taken into account.

The first approximation can be stated as follows if we assume that the parameter variation conforms to the normal distribution:

$$\sigma_C^2 = \sum_{i=1}^5 b_i^2 = \sum_{i=1}^5 \left(\frac{\delta C}{\delta x_i} \right) \sigma_i^2 \tag{14}$$

The application of the relationship gives us

$$\sigma_C^2 = \frac{S^2 h_1^2}{Ve^2} \sigma_{c1}^2 + \frac{S^2 C_1^2}{Ve^2} \sigma_{h1}^2 + \frac{S^2 h_2^2}{Ve^2} \sigma_{c2}^2 + \frac{S^2 C_2^2}{Ve^2} \sigma_{h2}^2 + \frac{S^2}{Ve^4} (C_1 h_1 + C_2 h_2) \sigma_{Ve}^2 \tag{15}$$

After simplifying this relationship (15), the following is obtained:

$$\sigma_{Ve} = \sqrt{\frac{Ve^4}{S^2(C_1 h_1 + C_2 h_2)} \sigma_C^2 - \frac{h_1^2 Ve^2}{(C_1 h_1 + C_2 h_2)} \sigma_{c1}^2 - \frac{C_1^2 Ve^2}{(C_1 h_1 + C_2 h_2)} \sigma_{h1}^2 - \frac{h_2^2 Ve^2}{(C_1 h_1 + C_2 h_2)} \sigma_{c2}^2 - \frac{C_2^2 Ve^2}{(C_1 h_1 + C_2 h_2)} \sigma_{h2}^2} \tag{16}$$

3.5. Application: Experimental Results

The parameters affecting the mixture’s concentration value have been defined in the section before:

- Ingredient Q1’s concentration in factor C1;
- Ingredient Q2’s concentration in factor C2;
- And factor Ve is the amount of water.

In our study, we consider that the parameter values are uncertain:

- The C1 and C2 concentrations rely on a number of factors and are determined to be inaccurate and unreliable [18];
- The quantity or weight of products Q1 and Q2 can be changed to alter levels h1 and h2;
- Depending on the values of C1 and C2, the amount of water to be added to Ve will vary [19];

We shall use the production’s actual statistical data to address this issue. The tolerances for these characteristics are then determined.

The standard deviation of the concentrates C, C1, and C2 and the amount of water Ve are provided to us by the process’s statistical data. These are, in order [13],

$$\sigma_c = 0.2 \quad \sigma_{c1} = 0.6 \quad \sigma_{c2} = 0.6; \quad \sigma_{Ve} = 0.5$$

We assume that the target values for good concentration are

- $m_1 = 200$ kg with C10 concentration = 0.25;
- $m_2 = 80$ kg with a concentration C20 = 0.07;
- Water volume: 650 L = Ve0.

It is stated that

- 1 m³ is the size of a mixing tank;
- S = 1 m² for the mixture tank section.

In these circumstances, the above equation may be used to calculate the standard deviation of the concentration:

ITE = 0.26 is the tolerance for water volume.

The following Table 1 lists the parameter tolerances [20]. Target final product C: concentrate of product. It is the target value the system aims to achieve.

Table 1. Target values and parameter limit values.

Parameter	Unit	Value
C1 min	g/mm ³	0.22
C1 max	g/mm ³	2.27
C2 min	g/mm ³	0.06
C2 max	g/mm ³	0.08
H min	m	0.2
H max	m	0.2
Ve min	m ³	0.8
Ve max	m ³	0.95
Cmin	g/mm ³	0.5
Cmax	g/mm ³	0.7
Target concentrate C1	g/mm ³	0.25
Target concentrate C2	g/mm ³	0.07
Target concentrate C	g/mm ³	0.6

The stocks at the system’s inputs are represented in this model by the location’s P01 and P02. The quantities of product C1 in reservoir 1, the level in reservoir 1, the quantity of product C2 in reservoir 2, and the level in tank 2 are, respectively, places PC1, Ph1, PC2,

and Ph2. The locations Pe and Pc stand for, respectively, the volume of water and the final mixing operation's concentration [21].

Figure 9 represents the network ICPN model of the mixing system (Figure 8). In this model, places P01 and P02 represent stocks to the entries of system. Places Pc1, Ph1, Pc2, and Ph2 are, respectively, the quantity of product C1 in tank 1, the level in tank 1, the quantity of product C2 in tank 2, and the level in tank 2. The places Pe and hc represent, respectively, the quantity of water and the concentration of the finished mixing operation. Place Pc is a place of control that prevents the simultaneous crossing of transitions T1 and T4.

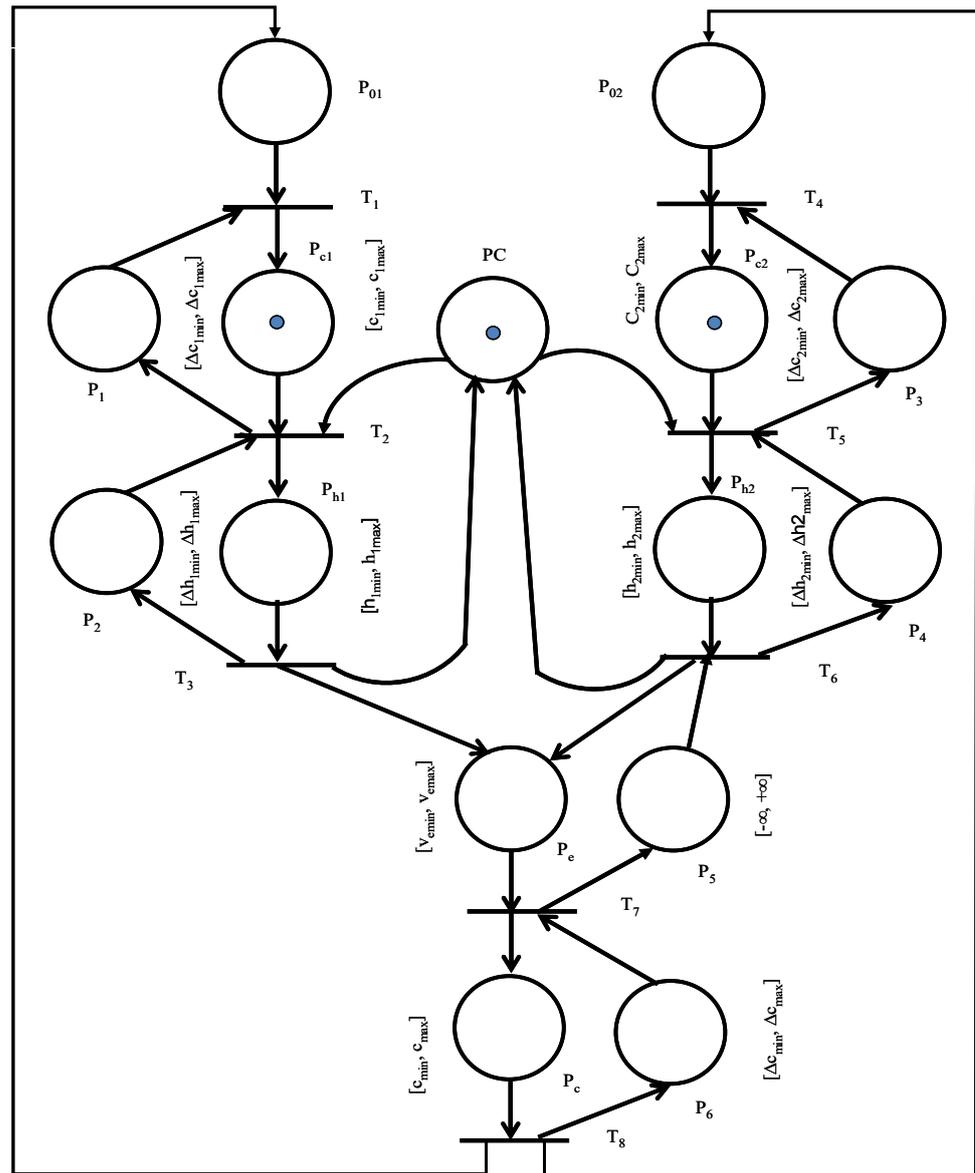


Figure 9. Fuzzy Interval RdP Model of the Mixing System.

Transitions T1 and T4 cannot occur at the same time, due to the control function of location Cp.

Finally, the process model is constructed, its structural properties may be examined, and it is demonstrated that the majority of P-time PN structural features can be applied to our model.

The method is used to determine interval limits using production data information [22].

4. Modeling the Process with Simulink

To simplify the problem, the behavior of the process is assimilated to a simple tank (Figure 10).

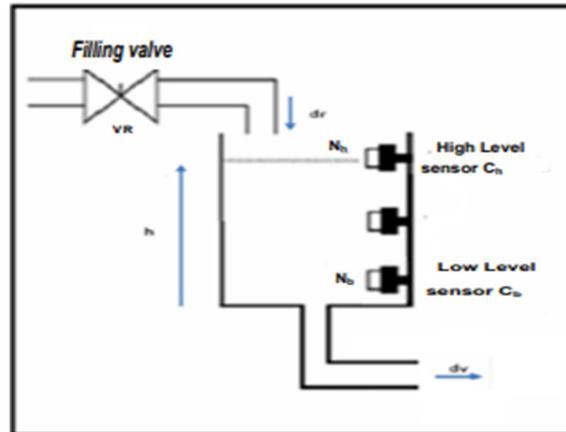


Figure 10. Simplified process diagram.

We consider three levels:

- N1: Low level (min limit);
- N0: Target level (optimal height);
- N2: High level (max limit).

The inputs and outputs considered in our process are summarized in Table 2.

Table 2. Process inputs/outputs.

	Entries		Outputs
N1	Low-level sensor	V3	Water filling valve in the mixing tank
N0	Medium-level sensor	VS	Mixed product flow valve
N2	High-level sensor		

Matlab’s Simulink module allows for simulating continuous, discrete and nonlinear systems in relation to the working memory of Matlab (workspace).

For the modeling of the mixing system, three blocks are considered:

- A control/command block for simulating an operating sequence.
- A process block to simulate the change in water level.
- A sensor block that provides all information on the level of mixture in the tank.

4.1. Level Modeling

The water level in the reservoir varies with inlet and outlet flows. The height in the tank varies according to the concentration of the product [23]. The value of the latter changes from batch to batch. We assume that these are small variations around an equilibrium point.

For the simulation, we take the following values:

- Tank section $S = 1 \text{ m}^2$;
- Min level: $N1 = 0.5 \text{ m}$;
- Max level: $N2 = 2.15 \text{ m}$;
- Optimal level: $N0 = 1.65 \text{ m}$;
- Input flow: $Q_i = 0.75 \text{ m}^3/\text{s}$;
- Output flow: $Q_s = 0.45 \text{ m}^3/\text{s}$.

Figure 11 shows the model (block diagram) of the level in the tank and the values adopted for the simulation.

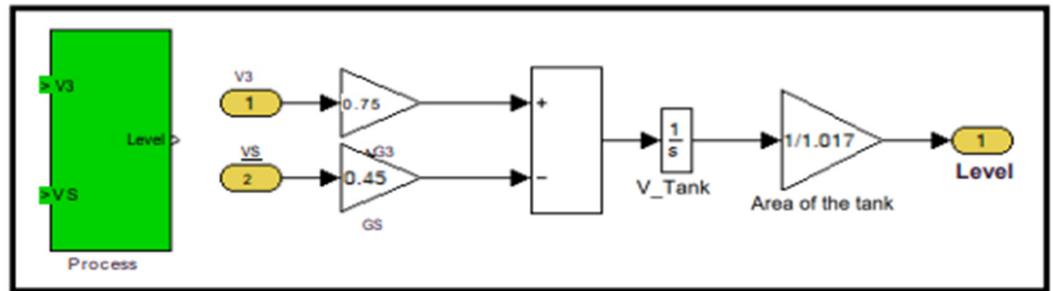


Figure 11. Block diagram of the tank level.

4.2. Modeling Level Sensors

The model of the levels is described in Figure 12. The level sensors N1, N0, and N2 are placed with the values given in the preceding paragraph [24].

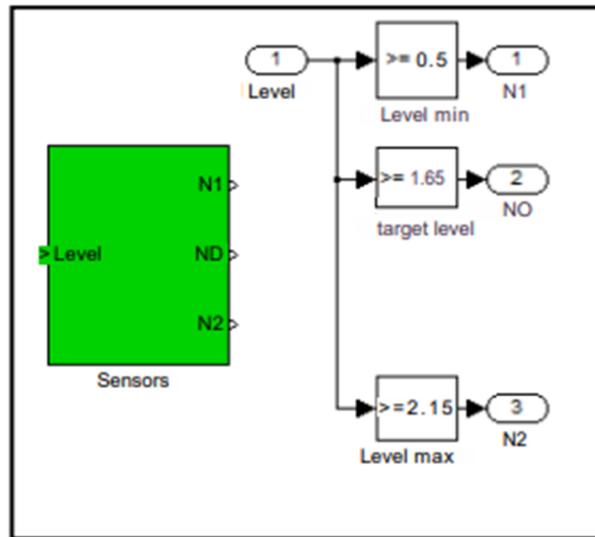


Figure 12. Tank level sensor modeling.

$N_i = 1$, if the level is reached;
 $N_i = 0$ otherwise.

4.3. Building the Global Model

For the construction of the overall model of the process, we consider the operating cycle described above. Figure 13 represents the overall model of the process under consideration.

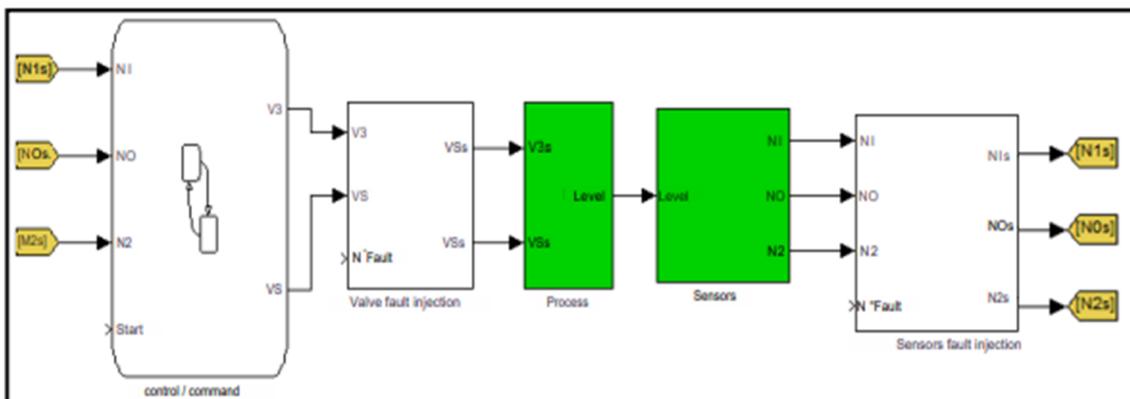


Figure 13. Overall fault injection process diagram.

5. Procedures for Constructing the Dynamic Model for Diagnosis

After constructing the overall model of the process (of Figure 8), which represents a part of the constructed fuzzy interval RdP model, in this section [25], we turn to the construction of the dynamic model for diagnosis [25]. This diagnosis must ensure the real-time detection and isolation of faults. To build our diagnosis, we follow the steps described in Figure 10.

- Step 1: Inventory probable failures. For this purpose, we use the FMEA method, and the results are summarized in Table 1.
- Step 2: Build the dynamic model of the system without faults. This allows us to perform the temporal identification of the process.
- Step 3: Build the dynamic model of the system in the presence of faults. This is a detection step.

This step is based on the time data summarized in Table 3.

Table 3. Dynamic behavior of the mixing tank.

	$\downarrow V_3$	h3 Height		
V3	[0.1, 57.2]	0.22	M	0.27
	[57.2, 60]	<0.22	>0.25	Fault
	[60, +∞]	Fault	Fault	Fault

- Step 4: Build the insulation block. In this step, the identification is based on the fuzzy rules and the defects considered.

6. Defect Modeling

Figure 14 shows the modeling of the injected defects of valve V3. In this model, we consider the defects following the request to open valve V3 ($V3s = 0$) and the request to close valve V3 ($V3s = 1$).

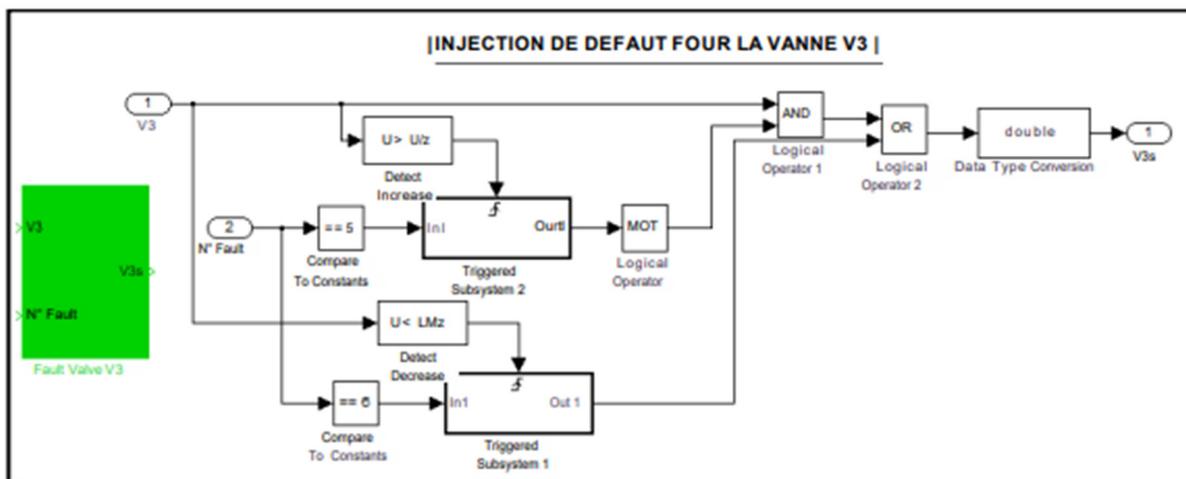


Figure 14. Modeling the defects of valve V3.

6.1. Simulation Results

Figures 15 and 16 illustrate, respectively, the injection of the defects of the opening request and of the closing of valve V3. We find that the valve remains closed despite the request for opening [26] and remains open despite the request for closing (Figure 16).

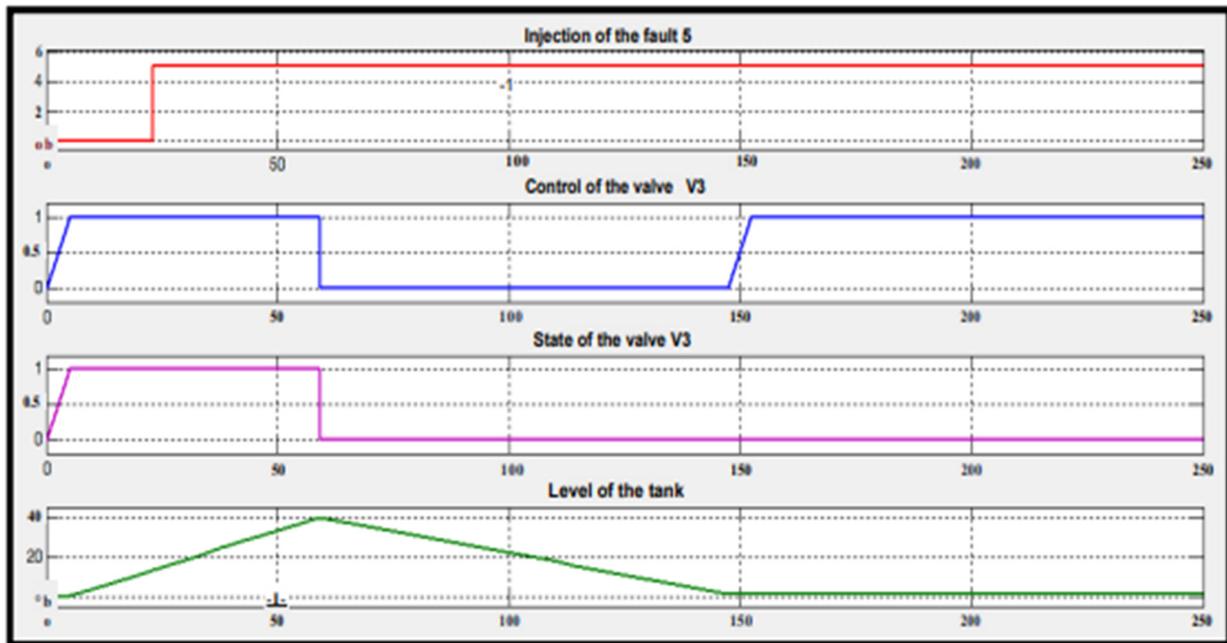


Figure 15. Valve V3 Stuck_Open.

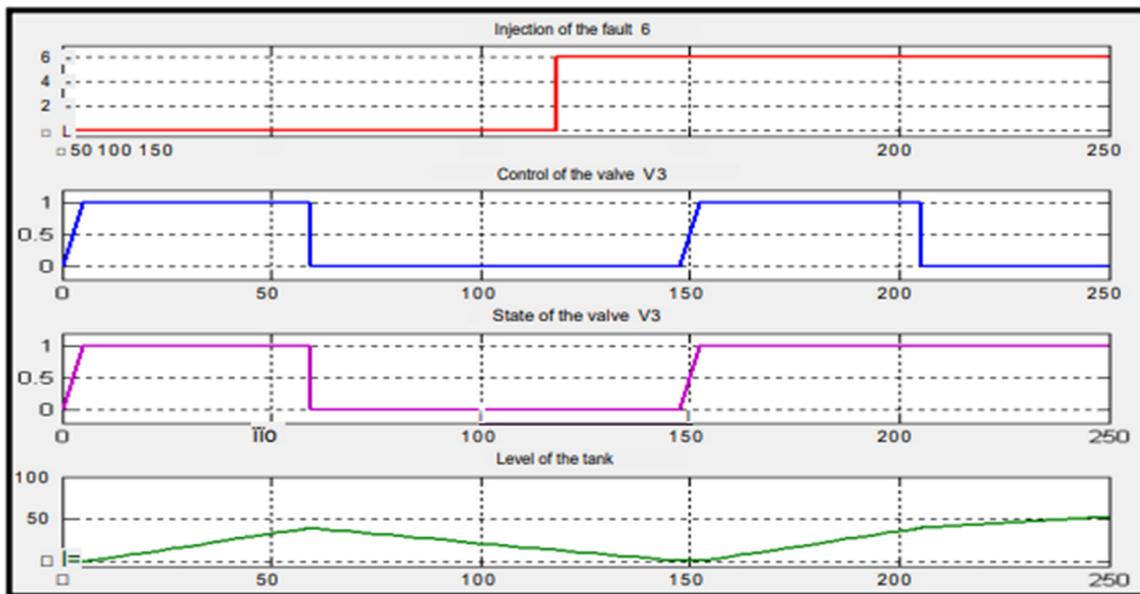


Figure 16. Valve V3 Stuck_closed.

6.2. Diagnostic Performance Analysis

In order to analyze the performance of the constructed diagnoser, we inject a defect into the system.

Figures 17 and 18 show, respectively, the normal operation of the method and the faulty operation. Figure 17 shows that, despite the request to open valve V3 (red signal), it remains closed (blue signal). This time represents the occurrence of a failure. Then, sensor N1 (purple signal) remains in state 0 at 3 s after the request to open the valve V3. This instant represents the instant of detection of the defect. Finally, sensor N0 (green signal) remains in the 0 state. After activation of the detection state, this instant corresponds to the location of the defect [27].

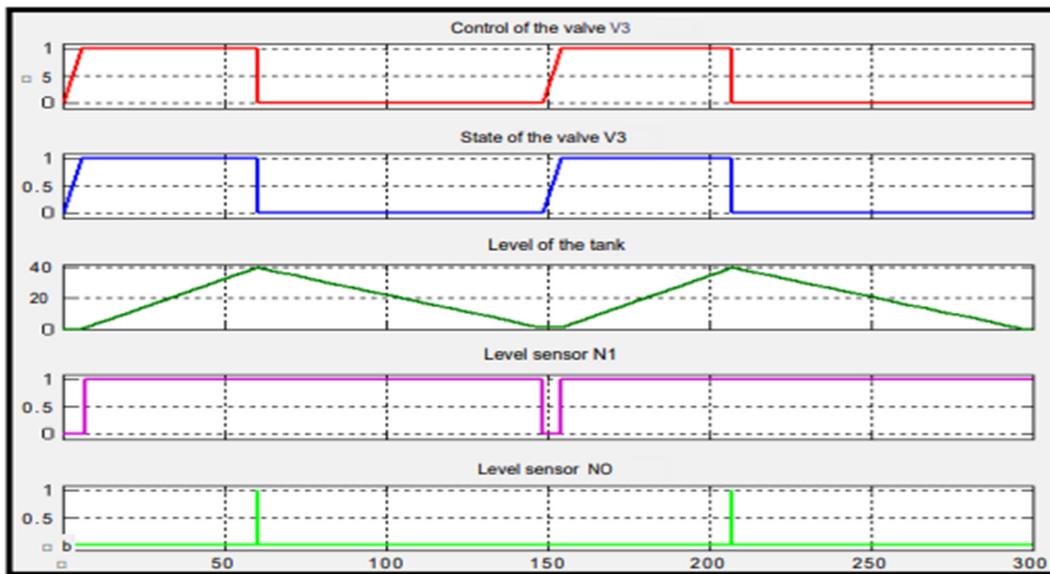


Figure 17. Normal functioning.

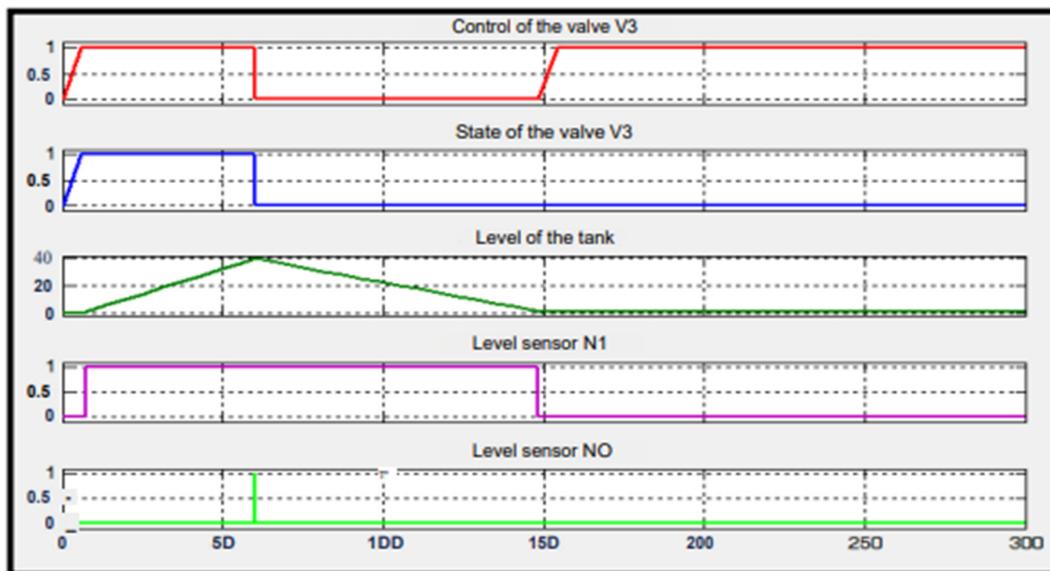


Figure 18. Diagnosis for the fault valve V3 Stuck_Close.

This analysis made by the diagnosis enables us to find the defect, and valve V3 remains closed at its opening request (V3_SC) (Figure 18).

7. Conclusions

In conclusion, the use of Petri’s networks in modeling manufacturing workshops has proven to be highly effective in representing the characteristics and interactions between different components. These networks are particularly useful in describing and solving complex problems, especially those involving time-constrained processes where adherence to specified constraints is crucial for product quality and conformity.

In this study, we presented an approach for automatic control specification of an industrial process, specifically focusing on maintaining a constant product concentration in the presence of non-deterministic concentrates C1 and C2. Our proposed control loop, based on an Interval Fuzzy Constraint Petri Net model, successfully achieved this objective.

The methodology employed in this research involved designing experiments to determine the validity intervals of critical parameters. Although a wide range of manual control

settings were generated, not all of them were considered, due to their variability. However, the Interval Fuzzy Constraint Petri Net model, incorporating completely defined intervals, provided a practical perspective by integrating experiential data into automation specifications. This approach combined Fuzzy Sets Theory, Statistics, and Petri Nets, allowing for a comprehensive analysis of the system.

The validity of our approach was demonstrated through its application to an industrial case study. While we chose the simplest case for validation purposes, it is important to note that our approach can be extended to more complex scenarios. For instance, it can be utilized in fault diagnosis involving common causes, utilizing the model's properties for supervision and diagnosis specifications.

Numerical characteristics of the obtained results, such as the accuracy of maintaining the desired product concentration under non-deterministic conditions, should be provided to further quantify the effectiveness and efficiency of the proposed approach.

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