Supplementary Material

Mathematical Models used in the SOFC [36].

Component	Equations	
Electrochemical reaction	$H_2 + 0.5O_2 \xrightarrow{yields} H_2O\ (T_{SOFC}.\Delta s + \Delta g)$	(S1)
Ideal efficiency	$\eta^0_{H_2} = \Delta g / \Delta h$	(S2)
Open Circuit	$U^{OCP} = \eta^0_{H_2} - \frac{R.T_{SOFC}}{2F} \ln\left[\frac{P_{H2O,TPB}}{P_{H2,TPB}.P_{O2,TPB}{}^{0.5}}\right]$	(S3)
Fuel cell potential	$U = U^{OCP} - (\eta_{ohm} + \eta_{conc} + \eta_{act,a} + \eta_{act,c})$	(S4)
Fuel Cell output power	$P_{SOFC} = U.j.L.W$	(S5)
Fuel cell electric efficiency	$\eta_{SOFC} = \frac{P_{SOFC}}{(\dot{n}_{CH4}LHV_{CH4} + \dot{n}_{H2}LHV_{H2} + \dot{n}_{CO}LHV_{CO})}$	(S6)
Fuel Utilization	$\eta_f = \frac{\Delta_{(in-out)}\dot{n}_{[H2+CO+CH4]}}{\dot{n}_{[H2+CO+CH4]\_in}}$	(S7)
Fuel cell overall efficiency	$\eta_O = \eta_f.\frac{\Delta g}{\Delta h}.\frac{U}{U^{OCP}}$	(S8)
Ohmic polarization	$\eta_{ohm} = j.R_{ohm} = j.\left(\frac{\tau_{anode}}{\sigma_{anode}} + \frac{\tau_{ele}}{\sigma_{ele}} + \frac{\tau_{cathode}}{\sigma_{cathode}}\right)$	(S9)
Concentration polarization	$\eta_{conc} = \frac{R.T_{SOFC}}{2F} \ln\left[\frac{P_{H2O,TPB}.P_{H2,f}}{P_{H2,TPB}.P_{H2O,f}}\right] + \frac{R.T_{SOFC}}{4F} \ln\left[\frac{P_{O2,a}}{P_{O2,TPB}}\right]$	(S10)
Anode activation polarization	$\eta_{act,anode} = \frac{R.T_{SOFC}}{Fn_e} \sinh^{-1}\left(\frac{j}{2j_{0,anode}}\right)$	(S11)
Cathode activation polarization	$\eta_{act,cathode} = \frac{R.T_{SOFC}}{Fn_e} \sinh^{-1}\left(\frac{j}{2j_{0,cathode}}\right)$	(S12)
Anode exchange density	$j_{0,anode} = \frac{R.T_{SOFC}}{Fn_e} k_{anode} \exp\left(-\frac{E_{anode}}{RT_{SOFC}}\right)$	(S13)
Cathode exchange density	$j_{0,cathode} = \frac{R.T_{SOFC}}{Fn_e} k_{cathode} \exp\left(-\frac{E_{cathode}}{RT_{SOFC}}\right)$	(S14)
The partial pressure of H2 at the triple phase boundary	$P_{H2,TPB} = P_{H2,f} - \frac{RT_{SOFC}\tau_{anode}}{2FD_{eff,anode}} j$	(S15)
The partial pressure of H_2O at the triple phase boundary	$P_{H2O,TPB} = P_{H2O,f} + \frac{RT_{SOFC}\tau_{anode}}{2FD_{eff,anode}} j$	(S16)
The partial pressure of O_2 at the triple phase boundary	$P_{O2,TPB} = P - (P - P_{O2,a}) exp \frac{RT_{SOFC}\tau_{cathode}}{4FD_{eff,cathode}P}.j$	(S17)
Electrochemical reaction heat released	$Q_{elec} = T_{SOFC}.\Delta s - j.(\eta_{ohm} + \eta_{conc} + \eta_{act,a} + \eta_{act,c})$	(S18)
Mass balance	$M_{i,in} + \sum_k C_{i,k} r_k = M_{i,out}$ Where $r_k = (j.N_{cells}.A)\ /\ (n_e\ F)$	(S19)

Energy balance equation in SOFC	$$\sum_{in} m_{in} cp_{in} T_{in} - P_{SOFC} = \sum_{out} m_{out} cp_{out} T_{SOFC}$$	(S20)