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A Pretest Estimator for the Two-Way Error Component Model

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Abstract: For a panel data linear regression model with both individual and time effects, empirical studies select the two-way random-effects (TWRE) estimator if the Hausman test based on the contrast between the two-way fixed-effects (TWFE) estimator and the TWRE estimator is not rejected. Alternatively, they select the TWFE estimator in cases where this Hausman test rejects the null hypothesis. Not all the regressors may be correlated with these individual and time effects. The one-way Hausman-Taylor model has been generalized to the two-way error component model and allow some but not all regressors to be correlated with these individual and time effects. This paper proposes a pretest estimator for this two-way error component panel data regression model based on two Hausman tests. The first Hausman test is based upon the contrast between the TWFE and the TWRE estimators. The second Hausman test is based on the contrast between the two-way Hausman and Taylor (TWHT) estimator and the TWFE estimator. The Monte Carlo results show that this pretest estimator is always second best in MSE performance compared to the efficient estimator, whether the model is random-effects, fixed-effects or Hausman and Taylor. This paper generalizes the one-way pretest estimator to the two-way error component model.

Keywords: two-way fixed-effects model; panel data; two-way random-effects model; two-way Hausman and Taylor estimator; Hausman test

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1. Introduction

For a panel data linear regression model with individual effects capturing heterogeneity, empirical studies select the random-effects (RE) estimator if the Hausman (1978) test based on the contrast between the fixed-effects (FE) estimator and the random-effects estimator is not rejected (see Owusu-Gyapong (1986) for one such example). Alternatively, they select the fixed-effects estimator in cases where this Hausman test rejects the null hypothesis (see Glick and Rose (2002) for one such example). The fixed-effects estimator allows *all* the regressors to be correlated with the individual effects, while the random-effects estimator assumes that *none* of the regressors are correlated with the individual effects (see Mundlak (1978) for an explanation of this *all*-or-*nothing* idea). Hausman and Taylor (1981) argued that not all regressors may be correlated with individual effects, and proposed an instrumental variable estimator called the Hausman and Taylor (HT) estimator, which uses both the between and within variation in the strictly exogenous variables as instruments. This estimator allows the estimation of the coefficients of time-invariant regressors which are wiped out by the FE estimator. The extra instruments are obtained using the individual means of the strictly exogenous regressors as instruments for the time-invariant regressors that are correlated with the individual effects. The choice of strictly exogenous regressors is tested using a second Hausman test based upon the contrast between the FE and the HT estimators. Examples of time-invariant regressors include the effect of distance on trade and foreign direct investment (see Egger and Pfaffermayr 2004), and the effect of common language on bilateral trade in a gravity equation (see Serlenga and Shin 2007). The effects of time-invariant variables like race and gender in a Mincer wage equation (see Cornwell and

Rupert 1998) are important in wage discrimination applications estimating the wage gap between males and females or black and nonblack people. Baltagi et al. (2003) proposed a pretest estimator for this one-way error component panel data regression model based on these two-Hausman tests. In fact, the standard Hausman (1978) test based on the contrast between the one-way RE estimator and the one-way FE estimator is applied first. If it is not rejected, the pretest estimator chooses the one-way random-effects estimator. But rather than accepting the one-way fixed-effects estimator in cases where this first Hausman test rejects the null hypothesis, a second Hausman test based on the difference between the one-way FE and the one-way HT estimators is performed. If this second Hausman test does not reject the null hypothesis, the pretest estimator chooses the one-way HT estimator. Otherwise, this pretest estimator chooses the one-way FE estimator. In this study, the Monte Carlo results show that this pretest estimator is always second best in MSE performance compared to the efficient estimator, whether the model is random-effects, fixed-effects or Hausman and Taylor.

This paper generalizes this pretest estimator to the two-way panel data linear model with individual *and* time effects. These could be macro-regressions of countries over time, or the marketing data of household purchases over repeated visits to a store. For the fixed versus random-effects in the two-way model, it is important to note that the Mundlak (1978) interpretation of the fixed-effects model as a correlated random effects model was generalized to this two-way model by Wooldridge (2021) and Baltagi (2023a). In fact, Baltagi (2023a) showed that in the Mundlak two-way model, the two-way fixed-effects model assumes that the time and individual effects are always correlated with *all* the regressors, whereas the two-way random-effects model assumes that they are uncorrelated with *all* the regressors. Once again, the choice between two-way fixed and two-way random-effects estimators is determined by a Hausman (1978) test, which was generalized from the one-way to the two-way model by Kang (1985).

Wyhowski (1994) generalized the Hausman and Taylor estimator from the one-way to the two-way model. Instead of all the exogenous variables being uncorrelated with the time and individual effects as in the two-way random effects model, or all the exogenous variables being correlated with the time and individual effects like in the two-way fixed-effects model, Wyhowski (1994) allows some but not necessarily all of the regressors to be correlated with the individual and time effects. Wyhowski (1994) assumes that the researcher knows which regressors are correlated with the time effects but not the individual effects, the regressors that are correlated with the individual effects but not the time effects, the regressors correlated with both time and individual effects, as well as the regressors that are not correlated with both effects. Baltagi (2023b), on the other hand, assumes that the researcher only knows which regressors are not correlated with both effects. Wyhowski's assumptions lead to more instrumental variables. These assumptions are testable using a Hausman-type over-identification test that is extended from the one-way to the two-way HT model (see Baltagi 2023b). The two-way HT estimator allows the estimation of the effects of time-invariant as well as individual-invariant regressors which are wiped out by the two-way fixed-effects estimator.[1]

In this study, Monte Carlo experiments are performed which compare the performance of this two-way pretest estimator with the standard panel data estimators under various designs. The estimators considered are ordinary least squares (OLS), two-way fixed-effects (TWFE), two-way random-effects (TWRE) and the two-way Hausman–Taylor (TWHT) estimators. In a two-way Hausman–Taylor design, we let some regressors be correlated with the individual effects and/or time effects. In a two-way RE design, the regressors are not allowed to be correlated with the individual and time effects. The Monte Carlo results show that the pretest estimator is always second best compared to the efficient estimator. It is second in RMSE performance compared to the two-way RE estimator in a two-way RE world, and second compared to the two-way HT estimator in a two-way HT world. The two-way FE estimator is a consistent estimator under both designs, but it is inefficient. The two-way HT estimator is the efficient estimator in the first design, and the two-way RE estimator is the efficient estimator in the second design. The disadvantage of the two-way FE estimator is that it does not allow the estimation of the coefficients of the time-invariant or individual-invariant regressors. Under the first design, where there is endogeneity among the regressors, we show that there is substantial bias in OLS and the two-way RE estimators, and both yield misleading inferences. Even under the second design, where there is no endogeneity between the time and individual effects and the regressors, inference based on OLS can be seriously misleading. This last result was emphasized by Moulton (1986).

Section 2 reviews the two-way Hausman and Taylor (1981) estimator first considered by Wyhowski (1994) and proposes a pretest estimator. Section 3 presents the Monte Carlo design, the results of the experiments and our conclusions.

2. The Two-Way Hausman and Taylor Estimator

Consider the two-way error component model:

$$y_{it} = \alpha + X_{it}'\beta + Z_i'\gamma + W_t'\delta + \mu_i + \lambda_t + \nu_{it} \quad i = 1, \ldots, N \text{ and } t = 1, \ldots, T \tag{1}$$

where y_{it} is the it-th observation on the dependent variable, α denotes the constant, X_{it}' represents $1 \times k$ time-varying as well as individual-varying regressors, Z_i' represents $1 \times g$ time-invariant regressors, and W_t' represents $1 \times h$ individual-invariant regressors. $\mu_i \sim IIN(0, \sigma_\mu^2)$, $\lambda_t \sim IIN(0, \sigma_\lambda^2)$ and $\nu_{it} \sim IIN(0, \sigma_\nu^2)$ independent of each other and themselves. let $n = NT$ denote the total number of observations.

In vector form, (1) can be written as

$$\begin{aligned} y &= \alpha\iota_{NT} + X\beta + Z\gamma + W\delta + Z_\mu\mu + Z_\lambda\lambda + \nu \\ &= V\Theta + Z_\mu\mu + Z_\lambda\lambda + \nu, \end{aligned} \tag{2}$$

where $y' = (y_{11}, \ldots, y_{1T}, y_{21}, \ldots, y_{2T}, \ldots, y_{N1}, \ldots, y_{NT})$ ordered by i as the slow index and t as the fast index. ι_{NT} is a vector of ones of dimension NT. X is $NT \times k$, Z is $NT \times g$, W is $NT \times h$. $V = [\iota_{NT}, X, Z, W]$ and $\Theta = [\alpha, \beta', \gamma', \delta']'$. $Z_\mu = I_N \otimes \iota_T$, $Z_\lambda = \iota_N \otimes I_T$, where $\otimes$ is the Kronecker product, I_N is an identity matrix of dimension N, ι_N a vector of ones of dimension N, $\mu' = (\mu_1, \ldots, \mu_N)$, $\lambda' = (\lambda_1, \ldots, \lambda_T)$, and $\nu' = (\nu_{11}, \ldots, \nu_{1T}, \nu_{21}, \ldots, \nu_{2T}, \ldots, \nu_{N1}, \ldots, \nu_{NT})$.

Wyhowski (1994) extended the Hausman and Taylor (1981) idea from the one-way to the two-way set up and allowed some but not necessarily all of the explanatory variables to be correlated with μ_i and λ_t. Wyhowski (1994) assumed that the researcher knows which Xs are correlated with μ_i but not λ_t, which Xs are correlated with λ_t but not μ_i, which Xs are correlated with both λ_t and μ_i, and which Xs are not correlated with both λ_t and μ_i. In this paper, we only know which Xs are not correlated with both effects. In particular, we consider the following model, where Z_i represents cross-sectionally variant but time-invariant variables, W_t are time-variant but cross-sectionally invariant variables, and X_{it}' is the it-th row of X. As in the one-way Hausman and Taylor (1981) model, we split the regressors X, Z and W into two sets of variables—$X = [X_1; X_2]$, $Z = [Z_1; Z_2]$ and $W = [W_1; W_2]$—where X_1 is $n \times k_1$, X_2 is $n \times k_2$, Z_1 is $n \times g_1$, Z_2 is $n \times g_2$, W_1 is $n \times h_1$, W_2 is $n \times h_2$ with $n = NT$, $k = k_1 + k_2$, $g = g_1 + g_2$ and $h = h_1 + h_2$. X_1, Z_1 and W_1 are assumed to be exogenous in that they are not correlated with μ_i, λ_t and ν_{it}, while X_2, Z_2 and W_2 are endogenous because they are correlated with μ_i or λ_t, but not ν_{it}. The two-way fixed-effects (FE) model or Within transformation would sweep the intercepts α, μ_i and λ_t and remove the bias, but in the process, it would also sweep the Z_i and W_t variables. Hence the two-way Within estimator will not give estimates of α, γ or δ. The two-way random-effects (RE) estimator assumes that the regressors are not correlated with the individual and time effects and applies a two-way random-effects GLS. A Hausman test based on the contrast between two-way FE and two-way RE determines whether two-way RE is efficient under the null hypothesis of no correlation between the regressors and the time and individual effects (see Kang 1985). Instead of this idea of "*all*" versus "*none*" of the regressors being correlated with the individual and time effects, the two-way

Hausman and Taylor (HT) estimator first proposed by Wyhowski (1994) allows some but not necessarily all of the regressors to be correlated with the individual and time effects. Assuming we only know which regressors are not correlated with both individual and time effects, Baltagi (2023b) proposed a modification of the Wyhowski (1994) estimator that uses fewer instruments and recovers the time-invariant as well as the individual-invariant variables which are important for policy studies. This is an instrumental-variables GLS estimator which can be implemented with a 2SLS or instrumental-variables regression after a two-way feasible GLS transformation due to Fuller and Battese (1974) (see the details in Wyhowski (1994) or Baltagi (2023b)). When both $N \to \infty$ and $T \to \infty$ and N/T is bounded, Wyhowski (1994) showed that the two-way Hausman–Taylor estimator is consistent. The two-way HT approach proposed by Baltagi (2023b) is summarized in the following Algorithm 1. A Hausman test based on two-way HT versus two-way FE determines whether the over-identification conditions are satisfied and, hence, whether the choice of exogenous X_1, Z_1, W_1 is rejected by the data (see Baltagi 2023b).

Algorithm 1 Estimation of a two-way Hausman–Taylor model

1. First step.
   (a) $\widetilde{\beta}_w = (X'Q_1X)^{-1}X'Q_1y$
   with $Q_1 = I_{NT} - P_\mu - P_\lambda + (\bar{J}_N \otimes \bar{J}_T)$
   with $P_\mu = Z_\mu(Z'_\mu Z_\mu)^{-1}Z'_\mu = I_N \otimes \bar{J}_T$ and $P_\lambda = Z_\lambda(Z'_\lambda Z_\lambda)^{-1}Z'_\lambda = \bar{J}_N \otimes I_T$
   and $\bar{J}_N = (\iota_N \iota'_N)/N$, $\bar{J}_T = (\iota_T \iota'_T)/T$.
   (b) $P_A = A(A'A)^{-1}A'$, $P_B = B(B'B)^{-1}B'$ with $A = [X_1, Z_1]$, $B = [X_1, W_1]$.
   (c) $\widehat{d} = Q_2\left(y - X\widetilde{\beta}_w\right)$, $\widehat{e} = Q_3\left(y - X\widetilde{\beta}_w\right)$ where $Q_2 = P_\mu - (\bar{J}_N \otimes \bar{J}_T)$
   and $Q_3 = P_\lambda - (\bar{J}_N \otimes \bar{J}_T)$.
   (d) $\widehat{\gamma}_{2SLS} = (Z'P_AZ)^{-1}Z'P_A\widehat{d}$ and $\widehat{\delta}_{2SLS} = (W'P_BW)^{-1}W'P_B\widehat{e}$.
   (e) $\widehat{\phi}_1 = \widetilde{y}'\bar{P}_{\widetilde{X}}\widetilde{y}/(N-1)(T-1)$ where $\widetilde{y} = Q_1y$, $\widetilde{X} = Q_1X$, $\bar{P}_{\widetilde{X}} = I_{NT} - P_{\widetilde{X}}$.
   (f) $\widehat{\phi}_2 = (y - X\widetilde{\beta}_w - Z\widehat{\gamma}_{2SLS} - W\widehat{\delta}_{2SLS})'Q_2(y - X\widetilde{\beta}_w - Z\widehat{\gamma}_{2SLS} - W\widehat{\delta}_{2SLS})/(N-1)$.
   (g) $\widehat{\phi}_3 = (y - X\widetilde{\beta}_w - Z\widehat{\gamma}_{2SLS} - W\widehat{\delta}_{2SLS})'Q_3(y - X\widetilde{\beta}_w - Z\widehat{\gamma}_{2SLS} - W\widehat{\delta}_{2SLS})/(T-1)$.
   (h) $\widehat{\phi}_4 = \widehat{\phi}_2 + \widehat{\phi}_3 - \widehat{\phi}_1$.
   (i) $\widetilde{\sigma}^2_v = \widehat{\phi}_1$, $\widetilde{\sigma}^2_\mu = (\widehat{\phi}_2 - \widehat{\phi}_1)/T$, $\widetilde{\sigma}^2_\lambda = (\widehat{\phi}_3 - \widehat{\phi}_1)/N$.
2. Second step.
   (a) $\widehat{\theta}_1 = 1 - \sqrt{\widetilde{\sigma}^2_v/\widehat{\phi}_2}$, $\widehat{\theta}_2 = 1 - \sqrt{\widetilde{\sigma}^2_v/\widehat{\phi}_3}$ and $\widehat{\theta}_3 = \widehat{\theta}_1 + \widehat{\theta}_2 + \sqrt{\widetilde{\sigma}^2_v/\widehat{\phi}_4} - 1$.
   (b) $y^* = y - \widehat{\theta}_1P_\mu y - \widehat{\theta}_2P_\lambda y + \widehat{\theta}_3Q_4y$ with $Q_4 = (\bar{J}_N \otimes \bar{J}_T)$.
   (c) Similarly, let $V^* = [\iota^*_{NT}, X^*, Z^*, W^*]$, with $\iota^*_{NT} = \left(1 - \widehat{\theta}_1 - \widehat{\theta}_2 + \widehat{\theta}_3\right)\iota_{NT}$.
   (d) $A_{HT} = [Q_1X, P_\mu X_1, P_\lambda X_1, Z_1, W_1, \iota_{NT}]$ and $P_{A_{HT}} = A_{HT}(A'_{HT}A_{HT})^{-1}A'_{HT}$.
   (e) $\widehat{\Theta}_{HT} = (V^{*\prime}P_{A_{HT}}V^*)^{-1}V^{*\prime}P_{A_{HT}}y^*$
   with $\widehat{\Theta}_{HT} = \left[\widehat{\alpha}_{HT}, \widehat{\beta}_{11,HT}, \widehat{\beta}_{12,HT}, \widehat{\beta}_{2,HT}, \widehat{\gamma}_{1,HT}, \widehat{\gamma}_{2,HT}, \widehat{\delta}_{1,HT}, \widehat{\delta}_{2,HT}\right]'$.
   (f) $\widehat{u}^*_{HT} = y^* - V^*\widehat{\Theta}_{HT}$ and $\widehat{\sigma}^2_{u^*_{HT}} = \widehat{u}^{*\prime}_{HT}\widehat{u}^*_{HT}/(NT - (k+g+h+1))$.
   (g) $Var\left[\widehat{\Theta}_{HT}\right] = \widehat{\sigma}^2_{u^*_{HT}}(V^{*\prime}P_{A_{HT}}V^*)^{-1}$.

For the two-way Hausman and Taylor model considered in (1), OLS is biased and inconsistent, while the two-way FE estimator which wipes out the intercept and the individual and time effects is consistent. The weakness of the fixed-effects estimator is that it also wipes out Z'_is and W'_ts, and therefore cannot estimate γ and δ. The two-way RE estimator is biased and inconsistent under the correlated random-effects two-way model of Hausman and Taylor. The two-way HT estimator is efficient under this model. In this case, the two-way pretest estimator performs the Hausman test for two-way FE versus two-way RE proposed by Kang (1985) and selects the two-way RE if the null hypothesis is not rejected. It then selects the two-way HT if it passes a second Hausman test based on two-way FE versus two-way HT. If this is rejected, the pretest selects the two-way FE estimator.[2]

In empirical applications, the two tests should be used successively, as shown in Figure 1. After calculating the TWFE ($\widetilde{\beta}_w$), TWRE ($\widehat{\Theta}_{fgls}$) and TWHT ($\widehat{\Theta}_{HT}$) estimators, the first test is the Hausman test defined by $\widehat{H} = \widehat{q}'\left(var(\widetilde{\beta}_w) - var(\widehat{\beta}_{fgls})\right)^{-1}\widehat{q}$, where $\widehat{q} = \widehat{\beta}_{fgls} - \widetilde{\beta}_w$, and $\widehat{\beta}_{fgls}$ is a subset of $\widehat{\Theta}_{fgls}$. Under H_0 (*none* of the individual effects or time effects are correlated with the regressors), $\widehat{H} \sim \chi^2_k$, and one chooses the efficient estimator TWRE. If this test is rejected, we use the over-identification test (called the Hausman–Taylor test in Figure 1) obtained by computing $\widehat{m} = \widehat{q}'\left(var(\widetilde{\beta}_w) - var(\widehat{\beta}_{HT})\right)^{\ominus}\widehat{q}$, where $\widehat{q} = \widehat{\beta}_{HT} - \widetilde{\beta}_w$, with $\widehat{\beta}_{HT}$ representing a subset of $\widehat{\Theta}_{HT}$, and $\ominus$ is the symbol of the generalized inverse. Under H_0, $\widehat{m} \sim \chi^2_l$ with $l = 2k_1 - g_2 - h_2$, and the efficient estimator TWHT is chosen. If the second test is rejected, the TWFE estimator is selected.

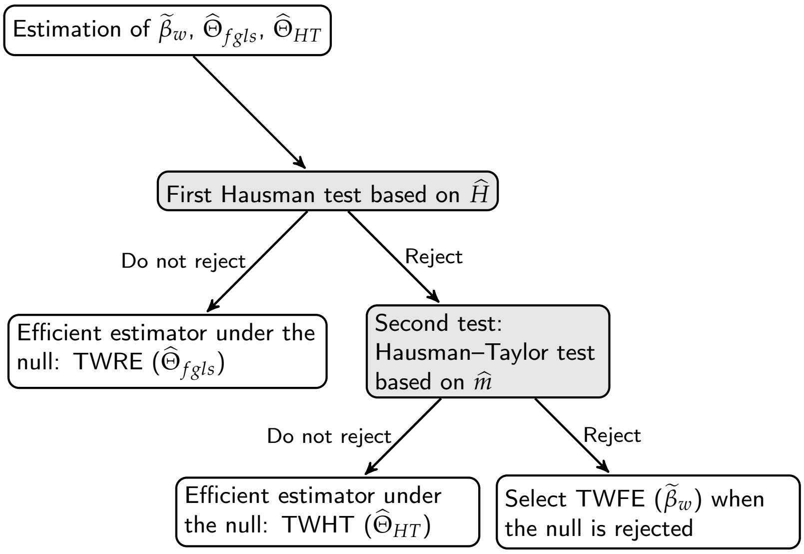

Figure 1. Pretest estimator.

The pretest estimator may be written as

$$\widehat{\Theta}_{pre} = I_{(0,c_1)}(\widehat{H})\widehat{\Theta}_{fgls} + I_{[c_1,\infty)}(\widehat{H}) \times \left[I_{(0,c_2)}(\widehat{m})\widehat{\Theta}_{HT} + I_{[c_2,\infty)}(\widehat{m})\widetilde{\beta}_w\right], \tag{3}$$

where $I_{(0,c_1)}(\widehat{H})$ and $I_{[c_1,\infty)}(\widehat{H})$ are indicator functions that take the values $I_{(0,c_1)}(\widehat{H}) = 1$ and $I_{[c_1,\infty)}(\widehat{H}) = 0$ if $\widehat{H}$ in the first Hausman test falls within the interval 0 and c_1 where c_1 is the 5% critical value for the χ^2_k statistics. This also means that $I_{(0,c_1)}(\widehat{H}) = 0$ and $I_{[c_1,\infty)}(\widehat{H}) = 1$ when $\widehat{H} > c_1$. Likewise, $I_{(0,c_2)}(\widehat{m})$ and $I_{[c_2,\infty)}(\widehat{m})$ are indicator functions that take the values $I_{(0,c_2)}(\widehat{m}) = 1$ and $I_{[c_2,\infty)}(\widehat{m}) = 0$ if $\widehat{m}$ in the second Hausman–Taylor test falls within the interval 0 and c_2 where c_2 is the 5% critical value for the χ^2_l statistics. This also means that $I_{(0,c_2)}(\widehat{m}) = 0$ and $I_{[c_2,\infty)}(\widehat{m}) = 1$ when $\widehat{m} > c_2$. It is clear from (3) that the pretest estimator is a function of the data, the hypothesis and the significance level of the two Hausman tests. As (3) is the sum of three parts, all three composed of products of non-independent random variables, and as underlined by Judge et al. (1978, 1988) and Giles and Giles (1993), to mention a few, the specification of the pretest estimator highlights the difficulty of deriving its sampling properties. And the choice of the significance level (here, 5%) of the χ^2 tests has a crucial role to play both in determining the proportion of use of each estimator and in determining the sampling performance of the pretest estimator.

3. Monte Carlo Results

Following Baltagi et al. (2003), we generalize the Monte Carlo design from the one-way to the two-way model:

$$y_{it} = \alpha + X_{1,it}\beta_1 + X_{2,it}\beta_2 + \gamma_1 Z_{1i} + \gamma_2 Z_{2i} + \delta_1 W_{1t} + \delta_2 W_{2t} + \mu_i + \lambda_t + \nu_{it} \tag{4}$$

where $X_{1,it} = [X_{11,it}, X_{12,it}]$, $\beta_1' = [\beta_{11}, \beta_{12}]$. $X_{1,it}$ is $(1 \times k_1)$ (here, $k_1 = 2$). Z_{2i} is $(1 \times g_2)$ (here, $g_2 = 1$). Z_{1i} and Z_{2i} are the time-invariant variables described below. W_{1t} and W_{2t} are the individual-invariant variables described below.

In our experiments, we set $\alpha = 5$, $\beta_1 = \beta_2 = \gamma_1 = \gamma_2 = \delta_2 = 1$, $\mu_i \sim IIN(0, \sigma_\mu^2)$, $\lambda_t \sim IIN(0, \sigma_\lambda^2)$ and $\nu_{it} \sim IIN(0, \sigma_\nu^2)$ independent of each other. The total variance across experiments is fixed at $\sigma^2 = \sigma_\mu^2 + \sigma_\lambda^2 + \sigma_\nu^2 = 3$. The proportion of variance due to individual effects $\rho_1 = \sigma_\mu^2/\sigma^2$ as well as the proportion of variance due to time effects $\rho_2 = \sigma_\lambda^2/\sigma^2$ is varied over the set (0.1, 0.2, 0.4, 0.6, 0.8) such that $(1 - \rho_1 - \rho_2)$ is always positive. We let $\rho_3 = \sigma_\nu^2/\sigma^2 = 0.1$, *i.e.*, $\sigma_\nu^2 = 0.3$. Then, $(\rho_1, \rho_2) = ((0.1, 0.8), (0.2, 0.7), (0.4, 0.5), (0.6, 0.3), (0.8, 0.1))$, to which we add the particular case $(\rho_1, \rho_2) = (0, 0)$ with $\sigma_\nu^2 = \sigma^2 = 3$. The (N, T) values considered are $(300, 100)$, $(200, 100)$ and $(300, 200)$. The number of replications is 1000.

The X_1 variables are generated following Nerlove (1971). For these series, the ratios of the between-individual (Bxx), between-time (Cxx) and the within-individual–time (Wxx) variabilities relative to the total variability are roughly 71%, 23% and 6%. Maddala and Mount (1973, p. 326) warned that for the two-way model, Wxx has to be small with respect to Bxx and Cxx; otherwise, the random-effects GLS would be equivalent to the fixed effects model, and the errors in the estimation of the variance components would not be of much consequence for estimating the slope coefficients. The $X_{1,it}$ variables are not correlated with μ_i and λ_t, and are generated as follows:

$$\begin{aligned} X_{11,it} &= 0.5X_{11,i,t-1} + \varphi_i + \phi_t + \zeta_{it} \\ X_{12,i,t} &= 0.5X_{12,i,t-1} + \vartheta_i + \tau_t + \xi_{it} \end{aligned} \tag{5}$$

where φ_i, ϑ_{it}, ϕ_t and τ_t are uniform on $[-4, 4]$, ζ_{it} and ξ_{it} are uniform on $[-2, 2]$, and Z_{1i} and W_{1t} are uniform on $[-3, 3]$.

We focus on the following two designs:

Case 1—A two-way Hausman–Taylor world, where X_2 is correlated with μ_i and λ_t by design, and Z_{2i} is correlated with μ_i as well as $X_{11,it}, X_{12,it}, X_{2,it}$. Also, W_{2t} is correlated with λ_t as well as $X_{11,it}, X_{12,it}, X_{2,it}$.

$$\begin{aligned} X_{2,it} &= 0.5X_{2,i,t-1} + \mu_i + \lambda_t + \vartheta_{it} \\ Z_{2i} &= \mu_i + \varphi_i + \vartheta_i + \chi_i \\ W_{2t} &= \lambda_t + \phi_t + \tau_t + \psi_t \end{aligned} \tag{6}$$

In the above equations, χ_i and ψ_t are uniform on $[-4, 4]$, and ϑ_{it} is uniform on $[-2, 2]$. Z_{2i} is correlated with $X_{11,it}$ by the common term φ_i, with $X_{12,i,t}$ by the common term ϑ_i and with $X_{2,it}$ by the common term μ_i. W_{2t} is correlated with $X_{11,it}$ by the common term ϕ_t, with $X_{12,i,t}$ by the common term τ_t and with $X_{2,it}$ by the common term λ_t.

Case 2—A two-way random-effects world, where Z_{2i} and W_{2t} are not correlated with μ_i and λ_t, but are still correlated with $X_{11,it}$ and $X_{12,it}$:

$$\begin{aligned} X_{2,it} &= 0.5X_{2,i,t-1} + \kappa_i + \varrho_t + \vartheta_{it} \\ Z_{2i} &= \varphi_i + \vartheta_i + \chi_i \\ W_{2t} &= \phi_t + \tau_t + \psi_t \end{aligned} \tag{7}$$

where κ_i and ϱ_t are uniform on $[-4, 4]$ and where $X_{2,it}$ is not correlated with μ_i and λ_t.

Table 1 shows the choice of the pretest estimator for various values of (ρ_1, ρ_2) in a Hausman–Taylor-type world when $N = 300$ and $T = 100$. For example, when $(\rho_1, \rho_2) =$

$(0, 0)$, out of 1000 replications, the pretest estimator chose the RE estimator in 946 replications, the HT estimator in 24 replications and the FE estimator in 30 replications. For $(\rho_1, \rho_2) \neq (0, 0)$, almost all replications chose the HT estimator. None selected RE, and between 9 and 14 replications selected FE. Note that as we vary (ρ_1, ρ_2), not only does the proportion of the total variance due to the random individual and time effects vary, but so does the extent of correlation between the regressors and the individual and time effects. For example, when $(\rho_1, \rho_2) = (0.2, 0.7)$, the mean correlation between $X_{2,it}$ and μ_i is 0.59. This rises to 0.84 when $(\rho_1, \rho_2) = (0.6, 0.3)$. In contrast, the mean correlation between $X_{2,it}$ and λ_t drops from 0.54 to 0.29 for these two cases. The mean correlation between Z_{2i} and μ_i is 0.19 and 0.32, and the mean correlation between W_{2t} and λ_t is 0.34 and 0.23 for these two cases. We focus on the coefficients of the endogenous regressors X_2, Z_2 and W_2, i.e., β_2, γ_2 and δ_2. The results of the other coefficients are available upon request from the authors. Table 1 reports the bias and RMSE (in %). When $(\rho_1, \rho_2) = (0, 0)$, OLS performs well in terms of bias and RMSE for all coefficients. When $(\rho_1, \rho_2) \neq (0, 0)$, HT, pretest and FE are the best in terms of RMSE for β_2, with HT and pretest performing the best for γ_2 and δ_2. Table 1 also reports the frequency of rejections in 1000 replications for $\beta_2 = 1$, $\gamma_2 = 1$ and $\delta_2 = 1$. This is assessed at the 5% significance level. Since the null hypothesis is always true, this represents the empirical size of the test. As expected, OLS performs badly, rejecting the null hypothesis when true in 99 to 100 percent of the cases, when $(\rho_1, \rho_2) \neq (0, 0)$. The same is true for the RE estimator since endogeneity is present. On the other hand, HT performs well, giving a size close to the 5% level. FE performs well for β_2, but it cannot estimate γ_2 and δ_2. The pretest performs well, with a size between 5% and 6% for β_2 and γ_2 and between 5% and 7% for δ_2.

Table 2 shows the choice of the pretest estimator for various values of (ρ_1, ρ_2) in a random effects-type world when $N = 300$ and $T = 100$. For example, when $(\rho_1, \rho_2) = (0.4, 0.5)$ out of 1000 replications, the pretest estimator is an RE estimator in 951 replications and an HT estimator in 49 replications. Now, there is no correlation between the regressors and the random individual and time effects. Table 2 also reports the bias and RMSE (in %) for β_2, γ_2 and δ_2. When $(\rho_1, \rho_2) = (0, 0)$, RE and OLS perform the best in terms of RMSE for all coefficients. The pretest is a distant third, while HT and the fixed-effects model perform poorly. When $(\rho_1, \rho_2) \neq (0, 0)$, in terms of RMSE, OLS performs poorly for all coefficients. RE is best, followed by the pretest and FE (only for β_2), and then, HT. For the frequency of rejections in 1000 replications for $\beta_2 = 1$, $\gamma_2 = 1$ and $\delta_2 = 1$ in an RE world, OLS is the only estimator that performs badly, rejecting the null hypothesis when true in a large percentage of cases, especially when $(\rho_1 \rho_2)$ are large. This is as large as 84% for β_2, 81% for γ_2 and 88% for δ_2.

Tables 3 and 4 consider the two-way Hausman and Taylor world and two-way RE world for $N = 200$ and $T = 100$, so that $N/T = 2$, rather than 3 in the case of Tables 1 and 2. Comparing Tables 3 and 4 to Tables 1 and 2, respectively, T remains fixed at 100, while N decreases from 300 to 200. Tables 5 and 6 fix N at 300 and double T from 100 to 200. By and large, similar rankings in terms of RMSE occur as described in Tables 1 and 2 but with different magnitudes.

As expected, holding T fixed at 100 and increasing N from 200 to 300, the RMSE of β_2 decreases for both the HT and RE worlds. In Table 3, the RMSE of the HT estimator of β_2 is of the order 0.296 to 0.300 for $(\rho_1, \rho_2) \neq (0, 0)$. This magnitude drops to the order 0.234 to 0.235 in Table 1 as N increases from 200 to 300, holding T fixed at 100. This RMSE range drops even further to $(0.171, 0.172)$ in Table 5 for $N = 300$ and $T = 200$, i.e., increasing both N and T. A similar decline in RMSE occurs for γ_2 for the HT estimator. The RMSE range is $(1.037, 2.817)$ in Table 1 ($N = 300$, $T = 100$), compared to $(1.272, 3.414)$ in Table 3 ($N = 200$, $T = 100$) and $(1.004, 2.792)$ in Table 5 ($N = 300$, $T = 200$).

Table 1. Hausman–Taylor world. Count, bias, RMSE, 5% size, $N = 300$, $T = 100$, 1000 replications.

		Count			Within β_2		
	(ρ_1, ρ_2)	RE	HT	FE	bias	rmse	size
1	$(0.0, 0.0)$	946	24	30	0.032	0.755	5
2	$(0.1, 0.8)$	10	979	11	0.004	0.235	5
3	$(0.2, 0.7)$	0	991	9	0.004	0.235	5
4	$(0.4, 0.5)$	0	986	14	0.004	0.235	5
5	$(0.6, 0.3)$	0	988	12	0.004	0.234	5
6	$(0.8, 0.1)$	0	989	11	0.004	0.234	5

	OLS									RE								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.032	0.754	5	0.007	0.359	5	-0.005	0.288	5	0.032	0.754	5	0.007	0.359	4	-0.005	0.288	4
2	44.926	44.953	100	4.166	4.276	100	8.996	9.224	100	19.410	19.423	100	2.832	2.908	100	11.238	11.497	100
3	44.370	44.387	100	4.147	4.249	100	8.194	8.421	100	27.408	27.419	100	4.022	4.076	100	9.495	9.734	99
4	43.411	43.418	100	4.224	4.306	100	6.457	6.687	100	34.561	34.569	100	5.107	5.142	100	7.028	7.253	94
5	42.513	42.516	100	4.426	4.482	100	4.546	4.770	100	37.857	37.863	100	5.623	5.650	100	4.929	5.135	86
6	41.590	41.592	100	4.745	4.776	100	2.445	2.610	99	39.752	39.757	100	5.922	5.946	100	2.878	3.015	83
	Hausman-Taylor									Pretest								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.032	0.756	5	-0.048	1.102	4	-0.025	0.801	4	0.032	0.755	5	0.003	0.494	5	-0.009	0.371	5
2	0.006	0.235	5	0.007	1.037	4	-0.038	5.819	6	0.182	1.752	6	0.050	1.096	5	0.048	5.924	7
3	0.006	0.235	5	0.012	1.432	5	-0.036	5.438	6	0.006	0.235	5	0.015	1.432	5	-0.060	5.443	6
4	0.006	0.235	5	0.019	2.002	5	-0.030	4.587	6	0.006	0.235	5	0.020	2.011	5	-0.048	4.593	6
5	0.006	0.234	5	0.025	2.444	6	-0.023	3.548	6	0.006	0.234	5	0.040	2.444	6	-0.020	3.552	6
6	0.006	0.234	5	0.029	2.817	5	-0.014	2.053	6	0.006	0.234	5	0.045	2.810	5	-0.020	2.057	6

bias $\times 10^{-2}$, rmse $\times 10^{-2}$.

Table 2. Random-effects world. Count, bias, rmse, 5% size, $N = 300$, $T = 100$, 1000 replications.

		Count			Within β_2		
	(ρ_1, ρ_2)	RE	HT	FE	bias	rmse	size
1	$(0.0, 0.0)$	949	27	24	0.025	0.737	5
2	$(0.1, 0.8)$	954	46	0	0.005	0.229	4
3	$(0.2, 0.7)$	951	48	1	0.005	0.229	4
4	$(0.4, 0.5)$	951	49	0	0.005	0.229	4
5	$(0.6, 0.3)$	948	52	0	0.005	0.229	4
6	$(0.8, 0.1)$	944	56	0	0.005	0.229	4

	OLS									RE								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	-0.008	0.182	4	0.008	0.360	5	-0.005	0.288	5	-0.007	0.182	4	0.007	0.360	4	-0.005	0.288	4
2	-0.044	1.468	82	-0.014	2.403	78	-0.048	4.140	88	0.004	0.219	4	-0.003	0.834	5	-0.032	4.019	6
3	-0.045	1.479	83	-0.019	2.489	78	-0.048	3.891	86	0.004	0.224	4	-0.002	1.158	6	-0.030	3.760	6
4	-0.045	1.494	82	-0.027	2.651	79	-0.046	3.334	85	0.004	0.227	4	0.000	1.620	5	-0.025	3.178	6
5	-0.042	1.505	84	-0.033	2.804	80	-0.041	2.660	82	0.004	0.228	4	0.001	1.977	6	-0.019	2.463	6
6	-0.036	1.510	84	-0.038	2.949	81	-0.033	1.733	76	0.004	0.227	4	0.002	2.279	5	-0.010	1.426	6

	Hausman-Taylor									Pretest								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.021	0.718	4	-0.049	1.101	4	-0.026	0.806	4	0.004	0.331	6	0.007	0.503	5	-0.006	0.375	5
2	0.005	0.229	4	0.005	1.037	4	-0.043	5.742	6	0.005	0.220	4	-0.007	0.878	6	-0.089	4.132	6
3	0.005	0.229	4	0.010	1.433	5	-0.041	5.372	6	0.004	0.224	4	-0.008	1.216	6	-0.061	3.879	6
4	0.005	0.229	4	0.017	2.004	5	-0.035	4.543	6	0.004	0.226	4	-0.008	1.698	6	-0.062	3.304	6
5	0.005	0.229	4	0.023	2.446	6	-0.027	3.524	6	0.004	0.227	4	-0.007	2.065	6	-0.053	2.570	6
6	0.005	0.229	4	0.027	2.820	6	-0.016	2.046	5	0.004	0.227	4	-0.016	2.363	6	-0.009	1.518	6

bias $\times 10^{-2}$, rmse $\times 10^{-2}$.

Table 3. Hausman–Taylor world. Count, bias, rmse, 5% size, $N = 200$, $T = 100$, 1000 replications.

		Count			Within β_2		
	(ρ_1, ρ_2)	RE	HT	FE	bias	rmse	size
1	$(0.0, 0.0)$	963	11	26	0.031	0.953	6
2	$(0.1, 0.8)$	38	947	15	0.031	0.299	5
3	$(0.2, 0.7)$	0	986	14	0.031	0.299	5
4	$(0.4, 0.5)$	0	987	13	0.031	0.298	5
5	$(0.6, 0.3)$	0	995	5	0.031	0.296	5
6	$(0.8, 0.1)$	0	988	12	0.031	0.295	5

	OLS									RE								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.029	0.950	6	0.017	0.436	4	-0.009	0.348	4	0.030	0.950	6	0.018	0.437	3	-0.009	0.348	4
2	44.928	44.956	100	4.200	4.321	100	9.093	9.311	100	19.345	19.362	100	2.849	2.960	100	11.369	11.612	100
3	44.361	44.379	100	4.177	4.290	100	8.289	8.507	100	27.313	27.328	100	4.046	4.123	100	9.619	9.846	99
4	43.389	43.397	100	4.249	4.339	100	6.545	6.764	100	34.465	34.476	100	5.138	5.189	100	7.134	7.350	96
5	42.482	42.485	100	4.446	4.511	100	4.619	4.833	100	37.771	37.779	100	5.658	5.697	100	5.014	5.214	87
6	41.549	41.551	100	4.763	4.802	100	2.492	2.654	99	39.676	39.682	100	5.955	5.988	100	2.931	3.069	81

	Hausman-Taylor									Pretest								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.031	0.953	6	0.059	1.360	5	0.025	0.987	5	0.030	0.950	6	0.031	0.523	4	-0.001	0.399	4
2	0.033	0.300	5	-0.050	1.272	5	0.082	5.680	6	0.712	3.491	9	0.131	1.431	9	0.525	6.070	10
3	0.034	0.299	5	-0.073	1.745	5	0.078	5.311	6	0.033	0.299	5	-0.072	1.749	5	0.064	5.325	6
4	0.034	0.298	5	-0.104	2.430	5	0.070	4.488	6	0.033	0.298	5	-0.106	2.432	5	0.080	4.487	6
5	0.033	0.297	5	-0.129	2.962	6	0.058	3.480	6	0.033	0.297	5	-0.125	2.962	6	0.056	3.485	6
6	0.033	0.296	5	-0.149	3.414	5	0.037	2.025	5	0.033	0.296	5	-0.138	3.424	5	0.033	2.028	5

bias $\times 10^{-2}$, rmse $\times 10^{-2}$.

Table 4. Random-effects world. Count, bias, rmse, 5% size, $N = 200$, $T = 100$, 1000 replications.

		Count			Within β_2		
	(ρ_1, ρ_2)	RE	HT	FE	bias	rmse	size
1	$(0.0, 0.0)$	946	26	28	0.028	0.926	5
2	$(0.1, 0.8)$	935	64	1	0.026	0.295	5
3	$(0.2, 0.7)$	937	62	1	0.026	0.295	5
4	$(0.4, 0.5)$	940	58	2	0.026	0.295	5
5	$(0.6, 0.3)$	941	59	0	0.026	0.295	5
6	$(0.8, 0.1)$	948	50	2	0.026	0.295	5

	OLS									RE								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.007	0.228	5	0.017	0.436	4	-0.009	0.347	4	0.008	0.228	5	0.017	0.437	3	-0.009	0.348	4
2	0.064	1.533	76	0.002	2.582	74	0.131	4.131	88	0.026	0.277	4	-0.025	0.983	5	0.107	3.935	6
3	0.069	1.574	77	0.003	2.734	76	0.131	3.893	88	0.027	0.285	5	-0.037	1.360	4	0.100	3.682	6
4	0.073	1.652	79	0.005	3.013	78	0.125	3.363	86	0.027	0.289	5	-0.053	1.900	5	0.084	3.114	5
5	0.073	1.725	79	0.006	3.266	80	0.113	2.724	83	0.028	0.290	5	-0.066	2.317	5	0.065	2.416	5
6	0.067	1.793	78	0.007	3.499	81	0.090	1.867	74	0.028	0.290	5	-0.076	2.669	5	0.038	1.405	5

	Hausman-Taylor									Pretest								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.028	0.894	5	0.058	1.363	5	0.021	0.995	5	-0.006	0.469	7	0.026	0.611	4	-0.003	0.463	4
2	0.026	0.295	5	-0.053	1.270	5	0.116	5.670	5	0.026	0.282	5	-0.039	1.035	5	0.091	3.996	6
3	0.026	0.295	5	-0.075	1.742	5	0.108	5.305	5	0.027	0.287	5	-0.059	1.421	4	0.061	3.755	6
4	0.026	0.295	5	-0.107	2.422	6	0.091	4.487	5	0.028	0.290	5	-0.075	1.970	5	0.069	3.222	6
5	0.027	0.295	5	-0.132	2.949	5	0.070	3.482	6	0.027	0.291	5	-0.070	2.394	5	0.074	2.514	6
6	0.027	0.295	5	-0.153	3.396	5	0.040	2.028	5	0.027	0.290	5	-0.081	2.715	5	0.045	1.499	6

bias $\times 10^{-2}$, rmse $\times 10^{-2}$.

Table 5. Hausman–Taylor world. Count, bias, rmse, 5% size, $N = 300$, $T = 100$, 1000 replications.

		Count			Within β_2		
	(ρ_1, ρ_2)	RE	HT	FE	bias	rmse	size
1	$(0.0, 0.0)$	959	19	22	0.012	0.527	4
2	$(0.1, 0.8)$	7	985	8	0.000	0.172	5
3	$(0.2, 0.7)$	0	993	7	0.000	0.172	5
4	$(0.4, 0.5)$	0	989	11	0.000	0.172	5
5	$(0.6, 0.3)$	0	985	15	0.000	0.171	5
6	$(0.8, 0.1)$	0	980	20	0.000	0.171	4

	OLS									RE								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.012	0.525	4	0.000	0.261	6	-0.009	0.208	6	0.012	0.525	4	0.000	0.261	6	-0.009	0.207	6
2	44.748	44.761	100	4.218	4.279	100	9.076	9.181	100	19.285	19.297	100	2.824	2.904	100	11.348	11.462	100
3	44.197	44.205	100	4.186	4.242	100	8.277	8.382	100	27.258	27.269	100	4.017	4.073	100	9.597	9.705	100
4	43.245	43.248	100	4.238	4.284	100	6.539	6.646	100	34.402	34.409	100	5.098	5.134	100	7.116	7.220	100
5	42.349	42.351	100	4.417	4.452	100	4.616	4.721	100	37.700	37.706	100	5.606	5.634	100	5.000	5.097	100
6	41.424	41.425	100	4.717	4.740	100	2.487	2.567	100	39.602	39.606	100	5.895	5.918	100	2.922	2.988	99

	Hausman-Taylor									Pretest								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.012	0.527	4	-0.031	0.765	6	-0.026	0.547	5	0.012	0.525	4	-0.006	0.366	7	-0.012	0.270	7
2	0.001	0.172	5	-0.049	1.004	5	0.077	3.918	5	0.121	1.443	5	-0.027	1.044	6	0.136	4.053	5
3	0.001	0.172	5	-0.062	1.404	5	0.072	3.663	5	0.001	0.172	5	-0.065	1.406	5	0.065	3.668	5
4	0.001	0.172	4	-0.080	1.976	5	0.061	3.094	4	0.001	0.172	4	-0.075	1.979	5	0.056	3.093	4
5	0.001	0.171	4	-0.094	2.418	5	0.047	2.395	5	0.001	0.171	4	-0.093	2.420	5	0.057	2.397	5
6	0.001	0.171	4	-0.105	2.792	5	0.024	1.386	4	0.001	0.171	4	-0.094	2.785	5	0.028	1.391	4

bias $\times 10^{-2}$, rmse $\times 10^{-2}$.

Table 6. **Random-effects world. Count, bias, rmse, 5% size, $N = 300$, $T = 100$, 1000 replications.**

		Count			Within β_2		
	(ρ_1, ρ_2)	RE	HT	FE	bias	rmse	size
1	$(0.0, 0.0)$	945	24	31	0.014	0.519	4
2	$(0.1, 0.8)$	949	49	2	-0.001	0.169	4
3	$(0.2, 0.7)$	947	51	2	-0.001	0.169	4
4	$(0.4, 0.5)$	955	45	0	-0.001	0.169	4
5	$(0.6, 0.3)$	959	41	0	-0.001	0.169	4
6	$(0.8, 0.1)$	960	39	1	-0.001	0.169	4

	OLS									RE								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	-0.003	0.133	4	0.001	0.261	6	-0.009	0.207	6	-0.004	0.133	4	0.001	0.261	6	-0.009	0.207	5
2	-0.039	1.102	82	-0.006	1.918	80	0.150	2.880	88	-0.002	0.165	4	-0.045	0.824	4	0.130	2.749	6
3	-0.045	1.161	83	-0.025	2.114	82	0.139	2.711	88	-0.002	0.167	4	-0.059	1.155	5	0.122	2.572	6
4	-0.051	1.269	84	-0.053	2.453	83	0.116	2.342	88	-0.002	0.168	5	-0.079	1.627	5	0.103	2.174	6
5	-0.055	1.366	86	-0.077	2.746	85	0.088	1.907	85	-0.002	0.168	4	-0.094	1.990	5	0.079	1.685	6
6	-0.056	1.453	86	-0.101	3.006	88	0.048	1.342	76	-0.002	0.168	5	-0.107	2.297	5	0.045	0.976	6

	Hausman-Taylor									Pretest								
	β_2			γ_2			δ_2			β_2			γ_2			δ_2		
	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size	bias	rmse	size
1	0.015	0.511	4	-0.029	0.765	5	-0.026	0.548	5	0.006	0.255	6	-0.001	0.358	7	-0.010	0.265	6
2	-0.001	0.169	4	-0.049	1.002	5	0.092	3.906	5	-0.002	0.165	4	-0.042	0.848	5	0.118	2.803	5
3	-0.001	0.169	4	-0.062	1.399	5	0.085	3.654	5	-0.001	0.167	4	-0.056	1.187	5	0.103	2.613	5
4	-0.001	0.169	4	-0.081	1.966	5	0.071	3.089	5	-0.001	0.168	5	-0.082	1.669	5	0.079	2.239	5
5	-0.001	0.169	4	-0.095	2.403	5	0.053	2.394	4	-0.001	0.169	4	-0.107	2.034	5	0.062	1.761	6
6	-0.001	0.169	4	-0.106	2.772	5	0.026	1.387	4	-0.002	0.168	5	-0.113	2.328	5	0.031	1.030	6

bias $\times 10^{-2}$, rmse $\times 10^{-2}$.

Similarly, for the RE estimator, the RMSE range for β_2 decreases from $(0.277, 0.290)$ in Table 4 (for $N = 200$, $T = 100$) to $(0.219, 0.227)$ in Table 2 (for $N = 300$, $T = 100$), and decreases further to $(0.165, 0.168)$ in Table 6 (for $N = 300$, $T = 200$). A similar decline in the RMSE happens for γ_2 for the RE estimator. The RMSE range is $(0.834, 2.279)$ in Table 2 ($N = 300$, $T = 100$), compared to $(0.983, 2.669)$ in Table 4 ($N = 200$, $T = 100$), and $(0.824, 2.297)$ in Table 6 ($N = 300$, $T = 200$). For δ_2, the RMSE performance improves as the N/T ratio declines. For the HT estimator, it is $(2.053, 5.819)$ for Table 1 ($N/T = 3$) and drops to $(2.025, 5.680)$ for Table 3 ($N/T = 2$) and $(1.386, 3.918)$ for Table 5 ($N/T = 1.5$). For the RE estimator, it is $(1.426, 4.019)$ for Table 2 ($N/T = 3$) and drops to $(1.405, 3.935)$ for Table 4 ($N/T = 2$) and $(0.976, 2.749)$ for Table 6 ($N/T = 1.5$).[3]

In summary, as in the one-way panel model, the OLS standard errors are biased and yield misleading inferences under both the two-way RE and HT worlds. RE, FE, HT and pretest yield the required 5% size under both designs for all values of (ρ_1, ρ_2). As expected, the RE estimator yields correct inference under a two-way RE world, but leads to misleading inference under a two-way HT world. In terms of bias, RMSE and inference, the pretest estimator is a viable alternative to two-way FE, RE and HT and should be considered in empirical panel applications.

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Notes

1. For an extension of the one-way Hausman and Taylor model to multidimensional panels, see Balazsi et al. (2017).
2. With large T panels, one may be concerned with serial correlation in the disturbances, and the Hausman and Taylor two-way estimator has to be modified to deal with this serial correlation.
3. These results corroborate assumptions 1 and 2 and theorem 2 of Wyhowski (1994) and in particular the important role of the constraint of a bounded N/T for asymptotic distributions. We also performed some robustness checks for alternative (N, T) combinations and alternative data generation processes for the exogenous and endogenous regressors.

References

Balazsi, Laszlo, Maurice J. G. Bun, Felix Chan and Mark N. Harris. 2017. Models with endogenous regressors, Chapter 3. In *The Econometrics of Multi-Dimensional Panels*. Advanced Studies in Theoretical and Applied Econometrics. Edited by Laszlo Matyas. Cham: Springer, vol. 50, pp. 71–100.

Baltagi, Badi H. 2023a. The Two-way Mundlak Estimator. *Econometric Reviews* 42: 240–46. [CrossRef]

Baltagi, Badi H. 2023b. The Two-way Hausman and Taylor Estimator. *Economics Letters* 228: 111159. [CrossRef]

Baltagi, Badi H., Georges Bresson, and Alain Pirotte. 2003. Fixed effects, random effects or Hausman-Taylor? A pretest estimator. *Economics Letters* 79: 361–69. [CrossRef]

Cornwell, Christopher, and Peter Rupert. 1988. Efficient estimation with panel data: An empirical comparison of instrumental variables estimators. *Journal of Applied Econometrics* 3: 149–55. [CrossRef]

Egger, Peter, and Michael Pfaffermayr. 2004. Distance, trade and FDI: A Hausman-Taylor SUR approach. *Journal of Applied Econometrics* 19: 227–46. [CrossRef]

Fuller, Wayne A., and George E. Battese. 1974. Estimation of linear models with cross-error structure. *Journal of Econometrics* 2: 67–78. [CrossRef]

Giles, Judith A., and David E.A. Giles. 1993. Pre-test estimation and testing in econometrics: Recent developments. *Journal of Economic Surveys* 7: 145–97. [CrossRef]

Glick, Reuven, and Andrew K. Rose. 2002. Does a currency union affect trade? The time series evidence. *European Economic Review* 46: 1125–51. [CrossRef]

Hausman, Jerry A. 1978. Specification tests in econometrics. *Econometrica* 46: 1251–71. [CrossRef]

Hausman, Jerry A., and William E. Taylor. 1981. Panel data and unobservable individual effects. *Econometrica* 49: 1377–98. [CrossRef]

Judge, George G., and Mary E. Bock. 1978. *The Statistical Implications of Pre-Test and Stein-Rule Estimators in Econometrics*. Amsterdam: Elsevier.

Judge, George G., R. Carter Hill, William E. Griffiths, Helmut Lutkepohl, and Tsoung-Chao Lee. 1988. *Introduction to the Theory and Practice of Econometrics*. New York: John Wiley & Sons.

Kang, Suk. 1985. A note on the equivalence of specification tests in the two-factor multivariate variance components model. *Journal of Econometrics* 28: 193–203. [CrossRef]

Maddala, Gangadharrao S., and Timothy D. Mount. 1973. A comparative study of alternative estimators for variance components models used in econometric applications. *Journal of the American Statistical Association* 68: 324–28. [CrossRef]

Moulton, Brent R. 1986. Random group effects and the precision of regression estimates. *Journal of Econometrics* 32: 385–97. [CrossRef]

Mundlak, Yair. 1978. On the pooling of time series and cross-section data. *Econometrica* 46: 69–85. [CrossRef]

Nerlove, Marc. 1971. Further evidence on the estimation of dynamic economic relations from a time series of cross sections. *Econometrica* 39: 359–82. [CrossRef]

Owusu-Gyapong, Anthony. 1986. Alternative estimating techniques for panel data on strike activity. *Review of Economics and Statistics* 68: 526–31. [CrossRef]

Serlenga, Laura, and Yongcheol Shin. 2007. Gravity models of intra-EU trade: Application of the CCEP-HT estimation in heterogeneous panels with unobserved common time-specific factors. *Journal of Applied Econometrics* 22: 361–81. [CrossRef]

Wooldridge, Jeffrey M. 2021. Two-Way Fixed Effects, the Two-Way Mundlak Regression, and Difference-in-Differences Estimator. Available online: https://www.researchgate.net/publication/353938385_Two-Way_Fixed_Effects_the_Two-Way_Mundlak_Regression_and_Difference-in-Differences_Estimators (accessed on 4 December 2023).

Wyhowski, Donald J. 1994. Estimation of a panel data model in the presence of correlation between regressors and a two-way error component. *Econometric Theory* 10: 130–39. [CrossRef]