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Didactical Materials Customizable to Suit Classroom Needs: A Valuable Resource for Teachers

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Abstract: Can free, adaptable, and didactically validated materials have an impact on teachers' practices and competences? This issue has been focused on by the researchers working on the *MaMa—Matematica per la scuola elementare* (MaMa—Mathematics for the primary school) project, commissioned by the Dipartimento dell'educazione, della cultura e dello sport (Department of education, culture and sport) of the Canton of Ticino (Switzerland). Since 2019, this project has been aiming to create materials for teaching and learning mathematics in primary school, in line with the curriculum. The innovative MaMa materials, which can be freely downloaded via the mama.edu.ti.ch platform, are addressed to both teachers and learners. In many cases, they are editable, so that they can be customized by each user to suit the different teaching contexts and pupils' learning needs and be grouped into collections. Via the administration and analysis of a questionnaire, this article investigates how teachers use the materials, and whether they influence teachers' practices and competences. The results of this pilot study show that MaMa materials are perceived from teacher–users as "materials for teacher education and development", especially at the disciplinary level, supporting both the instructional design process and the appropriation/transformation of didactical resources to deal with the challenges of differentiation in the classrooms.

Keywords: didactical materials; mathematics teaching–learning; resources accessibility; resources differentiability; resources use; teacher education and development

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1. Introduction

Nowadays, teachers are increasingly surrounded by a wide array of textual and digital resources from multiple sources, such as books, texts, course materials, and reports of school experiences. While this vast availability is on the one hand a valuable resource that enables teachers to increase their expertise, it also generates complexity in their work and increases their need to be able to select, integrate, and appropriate the materials they find. As a result, teachers' work increasingly requires competences related to conscious didactical planning, which also involve knowing how to identify materials that are valid from both disciplinary and didactical point of view. Moreover, such materials have to be in line with the curriculum, with institutional indications and with one's own didactical approach, and adaptable according to the peculiarities of the classroom context and the individual pupils' needs and inclinations. The teacher's design work thus entails the development of advanced reflective skills, which manifest themselves in the critical reading of the available resources and the adaptation of those selected to the context in which they operate.

These fundamental aspects of the active and creative documentation work of the teacher are even more evident in the Canton of Ticino (Switzerland), where the present study takes place, since at primary school level textbooks are traditionally neither provided nor used. Primary school teachers, therefore, are prompted to search for ideas and cues from external sources and to produce their own materials, encountering the difficulties of choice and selection highlighted in the reference literature [1–3]. To overcome these difficulties,

made explicit not only by the literature, but also by the teachers themselves and by the school heads, and in concurrence with the implementation of the *Piano di studio della scuola dell'obbligo ticinese* (Ticino compulsory school curriculum) [4] (henceforth referred to as *Piano di studio*), released in 2015 and revised in 2022, the Dipartimento dell'educazione, della cultura e dello sport (Department of education, culture and sport) of the Canton of Ticino felt the need to provide teaching materials for primary school teachers' use. This led to the creation of the *MaMa—Matematica per la scuola elementare* (MaMa—Mathematics for primary school) project, which focuses on the production of didactical materials in line with the curriculum and designed to support teachers' search and creation of mathematics resources. Such resources are intended to act as a "bridge" between the curricular indications and the classroom context, by supporting the teacher's design work while leaving ample room for choice and flexibility. On the one hand, teachers can draw from a large database of materials validated from a disciplinary and didactical point of view, and on the other hand, they can customize, modify, and adapt the materials according to their own needs and interests and to those of the class. A part of the MaMa materials has been published since December 2021 in the free dedicated platform https://mama.edu.ti.ch (accessed on 18 April 2024), while others are still in production.

The aim of this article is to investigate how teachers use these "bridging" materials and whether this use influences their disciplinary and didactical competences. In contrast to other studies carried out in this field with pre-service teachers (e.g., [2,5]), this article refers to in-service teachers in primary schools, among whom the integration into usual practices and an eventual change in knowledge and beliefs may encounter more resistance, but also be more significant [6–8]. In this sense, the present study aims to contribute to the existing ones by widening the field to the design that guides the teacher's appropriation of resources and make it always different, also depending on the classroom context and the pupils' skills.

2. Theoretical Framework

2.1. Didactical Engineering for Teacher Education and Development

This study can be ascribed within the theoretical framework of didactical engineering, which has been developed in the field of mathematics education since the 1980s and has evolved over time [9,10]. The Encyclopedia of Mathematics Education gives the following definition of didactical engineering, edited by Artigue [10] (p. 202):

> In the educational community, it mainly denotes today a research methodology based on the controlled design and experimentation of teaching sequences and adopting an internal mode of validation based on the comparison between the a priori and a posteriori analyses of these. However, since its emergence, the expression didactical engineering has also been used for denoting development activities, referring to the design of educational resources based on research results or constructions and to the work of didactical engineers.

In this article, we adopt the expression didactical engineering, in a broad sense, as the design and production of all or part of the curricular materials (e.g., a series of lessons, a lecture, a set of exercises, a textbook, computer software) while *necessarily* leaving a certain amount of didactical decision making to the teacher–user. In the most recent studies on this topic, Perrin-Glorian [11] and Perrin-Glorian and Bellemain [12] distinguish two types of didactical engineering:

- "First-generation didactical engineering", which consists of the production and dissemination of didactical resources designed by researchers in mathematics education to foster student learning, but which are often not adapted to the needs of ordinary teaching, i.e., to the needs of specific mathematics teaching–learning contexts.
- "Second-generation didactical engineering", also called "didactical engineering for teacher education and development", which aims to produce resources for ordinary teaching in classrooms, trying to take into account their functioning and teachers'

needs. This type of engineering aims to study the adaptation of produced situations to ordinary teaching conditions and needs in order to be of real support to teachers. As Perrin-Glorian and Bellemain [12] argue, this type of engineering "requires further study of the conditions under which it is implemented in the classroom and the ways in which teachers can manage the room for manoeuvre left by the situation" (p. 27, authors' translation) and contributes to teacher professional development and teaching improvement.

In the present study, we refer to the second type of engineering, the second-generation one, since we aim at fostering teacher professional development. In this sense, as Perrin-Glorian and Bellemain [12] point out, didactical engineering also allows the dissemination of results to teachers and the public, considering and studying didactical questions to meet and answer teachers' needs. Teachers' questions are thus addressed in two directions: towards research (increasing theory) and towards teaching (justifying choices of didactical transposition).

2.2. The Role of Instructional Design in Second-Generation Didactical Engineering

Instructional design plays a decisive role in second-generation didactical engineering since, as Perrin-Glorian and Bellemain [12] recall, citing Brousseau, the goal of didactical engineering is as follows:

> [. . .] to produce and test tools for the teacher's didactic action, enabling him or her to make reasoned and justified choices, asking "why" at every step and looking for verifiable answers (Brousseau, 2006, 1–12). The aim is not to provide teachers with ready-made solutions. (p. 20, authors' translation)

The fundamental elements of an instructional design that affect didactical engineering must therefore be considered. Among these, there are two intertwined classes of elements that influence the initial instructional design of a lesson or course [13]: on the one hand, the targeted goals, expected products, and curriculum topics; on the other hand, the teacher's perception of the learners' motivations and expectations, their knowledge and skills. The learning objectives and the learners' needs in relation to the expected products do indeed form the basis for the design of teaching and learning activities. However, the teacher's design work does not stop there, since it does not consist only in planning a learning experience that allows students to achieve objectives oriented by learning needs, which have been preliminarily hypothesized drawing on teacher's previous experience or detected through a diagnostic investigation of the context. It also presupposes "a profound relationship between subjects and contexts and an incessant recursiveness between design and action, according to [. . .] variables that occur directly in action, emerging from the instances proper to the environment or to the actors participating in the teaching process" [14] (p. 131, authors' translation).

In particular, there is a relationship between the space of teachers' academic knowledge ("First space") and the space of students' cultural knowledge ("Second space") that gives rise to a so-called "Third space", where teachers' culture and academic knowledge meet with students' culture and everyday knowledge [15]. The continuous dialogue and feedback between teaching and learning supports the development of the "Third space" by offering teachers an opportunity to interweave research theories and perspectives with reflective practices. In light of this, taking up Pane's definition [15] of the three types of spaces, Pentucci [14] highlights how the teacher–designers, confronted with chaotic, complex, and multidimensional situations, typical of educational realities, must be flexible in the revision of their own project. They must indeed accept the instances coming from the context and proceed by hypothesis, attempts, and sometimes, inevitably, errors.

Even Laurillard [13], with whom we opened this section, emphasizes that instructional design for learning is not an exact science, but that it needs a continuous exchange of ideas and experiences to generate knowledge in the field: "Teaching is a 'design science' closer to engineering than to physics because it is explicitly aimed at producing change towards

an expected outcome" (p. 109, authors' translation). Given the complex, uncertain and non-deterministic nature of instructional design for learning, teachers must themselves be learners, generating constant professional development. And this occurs through self-critical rereading and continuous revision of what is proposed, as a result of the incessant feedback loops given by the interactive nature of teaching–learning process.

2.2.1. Instructional Design as a Competence-Building Workshop

Teaching as a design science and the image of the teacher as a didactical engineer evoke theories of the "laboratory as a place of competence building" [16]. Teacher–designers can be imagined as engineers in their "teaching laboratory", surrounded by resources and artefacts, with which they create, revise, and refine their design of a lesson or series of lessons.

In this context, teachers develop competences through a laboratory dimension, as "the identity of the laboratory is structured around the nature of the learning that takes place in it and the type of processes that generate it" [16] (p. 71, authors' translation). The teacher's design work, in a laboratory dimension, stems from the subject's interaction with the environment that is susceptible to representations and interpretations. As Grange [16] points out, each teacher chooses the resources to be mobilized according to his/her own representations of the situation. Moreover, to successfully deal with the initial situation, it is also necessary to organize, coordinate and operate on the selected resources, in a process-oriented approach, which is "focuses on the way competence is originated, enhances learning and responsibility, hence the educational dimension and conscious experience, beyond the "doing", and opens to reflectiveness as a characteristic property of [teachers'] expertise" (pp. 71–72, authors' translation).

Such a laboratory dimension of design must therefore be facilitated by resources that, on the one hand, help to manage the complexity of the context and the didactical action and, on the other hand, suggest both constructive and reflective operations, guiding the teacher–designer in the delicate process of didactical transposition of the mathematical knowledge at stake [14].

2.2.2. The Documentational Approach to Didactics with a Focus on Differentiation

In the design context outlined, the "resources" available or retrievable by the teachers are to be conceived in Adler's sense [17] (p. 207): "to think of a resource as the verb 're-source' suggests, to nourish again or in a different way". Based on this assumption, the documentational approach to didactics was developed in order to study teachers' professional development by analyzing their interactions with the resources they use and construct for the needs of their teaching [1,3].

As pointed out by Pepin and colleagues [18], and echoed by Jones and Pepin [19], the teacher's interaction with a resource can refer to one or more of the following processes:

- *Adoption/integration*: the literature refers to these processes when the focus is on why a teacher chooses to use or not to use a certain resource. In particular, authors speak of *adopting* a textual resource (e.g., a textbook [20]) or *integrating* a digital resource (e.g., a software [21]), considering two fundamental conditions: the potential of a resource to be included in the teacher's usual teaching practices and its compatibility with the teacher's view of mathematics and its teaching.
- *Documentational genesis/appropriation*: this is a two-way process from the teacher to the resource and vice versa, in which the teacher's beliefs and practices shape his or her use of the resource, and at the same time the characteristics of the resource may contribute to the teacher's professional development and thus generate an evolution in his or her beliefs and practices [1].
- *Transformation*: in the literature, this term is used to refer to the interaction of teachers with curricular materials designed, produced, and disseminated specifically to promote teacher learning. An example is the so-called "educative curriculum materials" described by Davis and Krajcik [22], such as materials that support teachers in their

instructional design, which must maintain a good balance between guidance and choice in order to be effective.

Jones and Pepin [19] point out that one of the most important learning processes is precisely that of *transformation*, in which teachers develop their design competences while appropriating curriculum principles and materials.

Research on teachers' use of curriculum materials (e.g., [23,24]) suggests that there are significant variations in how teachers use these resources, depending on their knowledge and beliefs about mathematics, about students, and about how students learn as well as on teachers' orientations towards the curriculum [25]. Remillard and colleagues [24] show how different uses of the same resource imply very different design characteristics and can substantially influence students' learning opportunities. The teacher's *appropriation* of a resource, in its bidirectional nature (from the teacher to the resource and vice versa), is mostly an active and creative operation that is fundamental to adapt the material to one's own teaching approach, and also to differentiate it according to the needs of the classroom context and of individual learners. Differentiation is in fact an important aspect of the act of appropriating a resource and, the more flexible and adaptable the resource is to different contexts, the greater the possibilities of differentiating it. This is a crucial feature from the perspective of universal design for learning, which suggests designing flexible and customizable pathways that can be effective for all learners, so as to allow a shift from individualized differentiation to a priori differentiation [26].

3. Context and Research Question

3.1. The MaMa Project

In 2019, the Dipartimento dell'educazione, della cultura e dello sport (Department of education, culture and sport) of the Canton of Ticino commissioned the head of the Competence center for mathematics education of the Department of education and learning/Universities of Teacher Education of the University of Applied Sciences and Arts of Southern Switzerland located in Locarno (Switzerland) to design, produce and disseminate innovative didactical materials for the teaching–learning of mathematics from first to fifth grade. The request was that the commissioned materials be consistent with the curriculum and take into account the needs expressed by teachers and school heads. This is how the *MaMa—Matematica per la scuola elementare* (MaMa—Mathematics for primary school) project was born, a project designed to create resources aimed at both teachers and pupils, to incorporate the results of mathematics education research and at the same time effectively bring the world of research closer to the world of schools. In fact, it is known that "when engineering is validated from a research point of view, with a good control of variables, as it allows certain knowledge to emerge in pupils, it is not necessarily validated for its dissemination in ordinary teaching" [12] (p. 27).

The intention was therefore to create a "second-generation didactical engineering", through the production and diffusion of materials which had been already extensively experimented and tested in classrooms and used in teacher education courses both in Ticino and in Italy. At the same time, such materials had to be varied, rich and easily customizable by teachers to be easily adaptable to each actual classroom context.

To operationalize this intention, a working team was created consisting of diverse and complementary figures, i.e., not only researchers in mathematics education but also teachers and institutional figures such as headmasters and inspectors, so as to create a good connection between the institutional bodies in charge of education at the various school levels and the world of research in mathematics education.

The project tagline became "*MaMa: of all, for all*". *Of all*, because the materials were produced not only starting from the proposals of the project team members and those validated by scientific research in mathematics education, but also taking into account the effective practices and traditions of the Ticino region, thanks to the contribution of numerous teachers who wished to share their own "good practices". *For all*, because the materials created, in addition to being freely accessible by visiting the dedicated platform

(mama.edu.ti.ch), can be customized by each user to best suit the different teaching contexts, thus favoring an a priori differentiation for the whole class, but at the same time also an individualized differentiation to meet the learning needs of individual pupils, each one unique and different from the others.

3.1.1. The Different Types of MaMa Materials

A variety of materials designed for both the teacher and the pupil has been produced. For teachers, there are the *guidelines*, in which it is possible to trace the methodological choices underpinning the project, in-depth studies of the various mathematical topics, various didactical aspects to be considered when dealing with these topics and some historical background. Also intended for teachers are the *contexts of meaning*, conceived as a support for dealing with real problem situations with children in which to encounter, apply and learn mathematics; the *didactical practices*, in which activities are presented to be proposed to pupils in laboratory mode [27,28]; different types of *problems* to be tackled in the classroom using different approaches; *games*, accompanied by all the material needed to propose them in the classroom (Figure 1a); and various *supports* for teachers, i.e., concrete tools such as coins, banknotes, developments of geometric figures, and much more, which can be useful to implement the proposed activities in the classroom (Figure 1b).

(**a**) (**b**)

Figure 1. Examples of MaMa materials: (**a**) the "Battle cards" game; (**b**) the "Coins and banknotes" support.

All these materials have been formatted with eye-catching graphics and made into free editable PDF documents, to be customized by teachers with a simple mouse click.

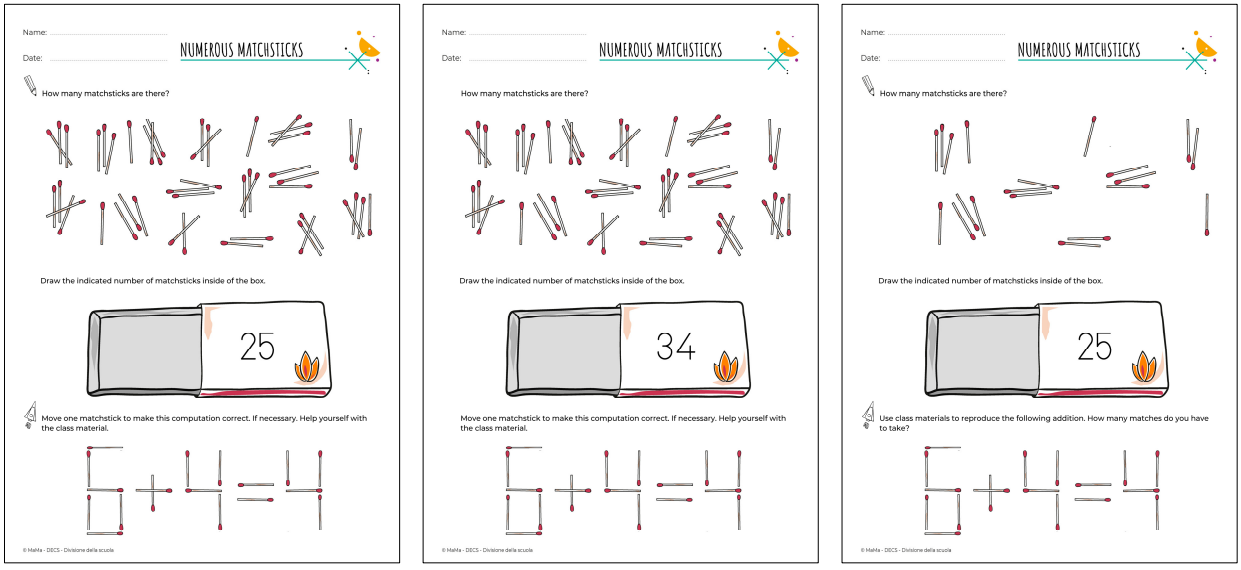
Figure 2 shows some cards of the "Mathematical taboo", an adaptation of the famous "Taboo" where a player must describe a given word to the other players but without using a list of prohibited words. Two versions of "Mathematical taboo" cards are provided as an example here: the version that is directly downloadable online (with for instance the card "Time" and the prohibited words: measure, clock, hours, minutes, seconds, sandglass) and one possible edited version with fewer prohibited words to simplify the description (with for instance the card "Time" and the prohibited words: clock, hours, minutes, seconds).

Furthermore, numerous *worksheets* are available for pupils to introduce, accompany and consolidate learning of the various mathematical contents. Each worksheet can be customized and differentiated by the teacher by modifying the texts, the tasks, the numbers involved, and sometimes even the illustrations, in order to adapt to each pupil's learning needs and skills. In Figure 3 the worksheet for second grade entitled "Numerous matchsticks" is proposed in three different versions: the first is the original one available on the platform, the second is obtained by switching off the icons related to prevailing competence aspects and changing the numerical information, while the third is obtained by

playing on the number of illustrations, their spatial arrangement and the overlapping of the matchsticks, so as to facilitate pupils with spatial perception difficulties. Figure 4, instead, shows two versions of the "Treasure map" worksheet for fourth grade: the first on the left is the original one, while the second is obtained by varying the texts and the numbers to be compared, so as to meet the skills of a group of lower achievers in the same class.

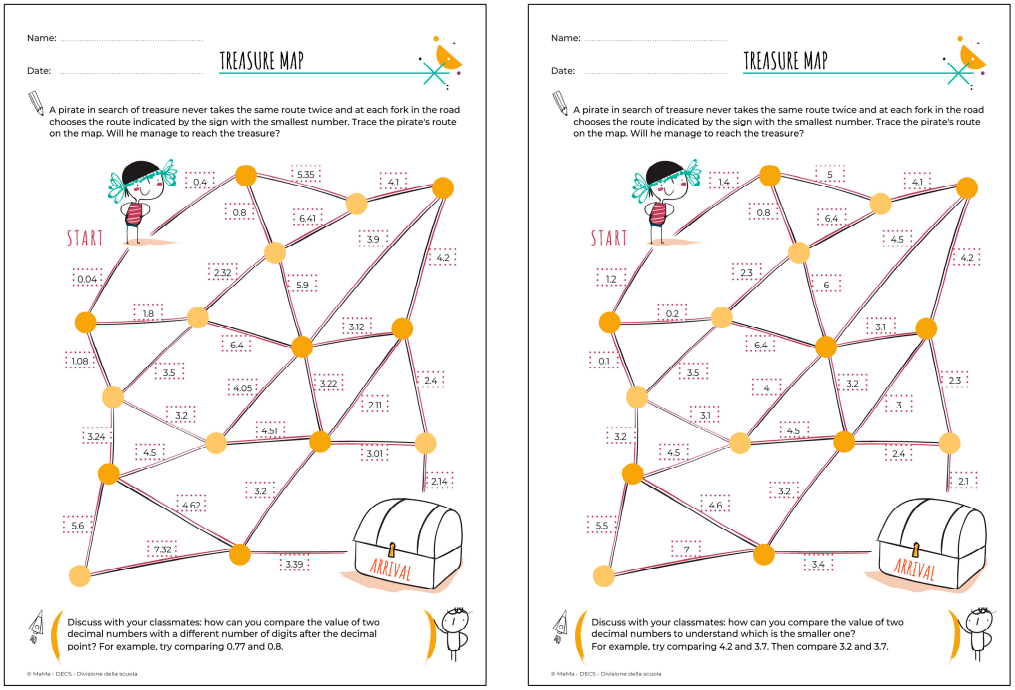
(**a**) (**b**)

Figure 2. Two versions of the "Mathematical Taboo" game cards: (**a**) original version with six forbidden words; (**b**) edited version with four forbidden words.



(**a**) (**b**) (**c**)

Figure 3. Three versions of the "Numerous matchsticks" worksheet for second grade: (**a**) original version; (**b**) edited version without icons of competence aspects and one numerical information changed; (**c**) edited version with no overlapped matchsticks and one task changed.



(**a**) (**b**)

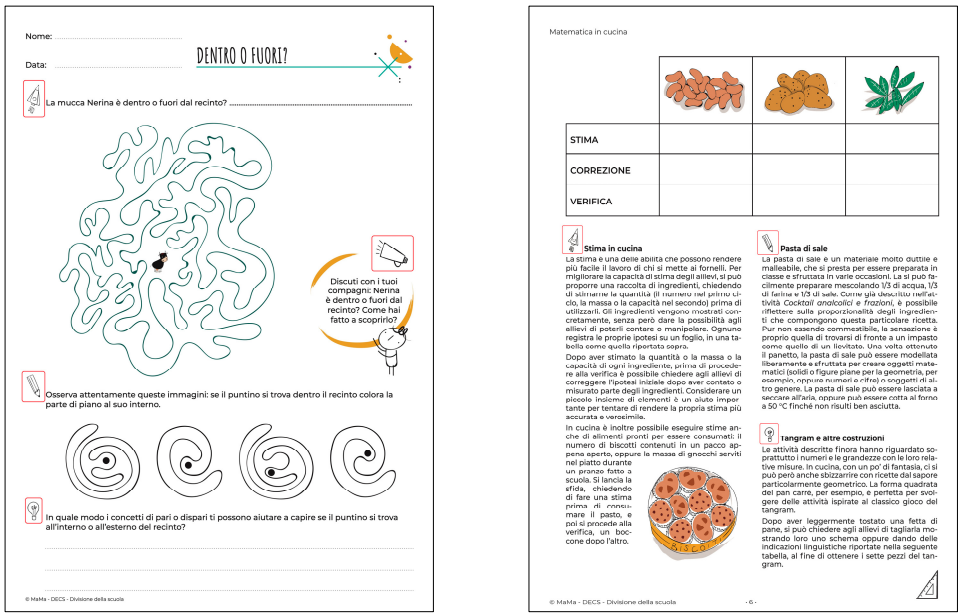
Figure 4. Two versions of the "Treasure map" worksheet for fourth grade: (**a**) original version; (**b**) edited version with numbers and the last task changed.

3.1.2. The Features of MaMa Materials

The peculiar features of MaMa materials can therefore be summed up as follows:

- *Plurality*: the proposals for teachers are described at several levels. Some indications are given at a macro level, such as the disciplinary indications in the guidelines or the didactical prompts in the contexts of meaning or didactical practices, into which teachers can freely graft real problem situations; other indications are provided more at a micro level, such as specific tasks and tools provided in games, supports, and worksheets that the teacher can however decide how and when to propose.
- *Vastness*: the materials are many, specifically designed to create a very extensive database of resources that teachers must necessarily select according to their own needs and those of the class.
- *Validity*: all materials are produced, reviewed, and validated by a team of disciplinary and educational experts, written in a linguistic font that meets the criteria of high readability, and accompanied by specially produced graphic illustrations for a layout that is not only attractive, but also functional for understanding the text.
- *Consistency with the curricular indications*: all materials are designed to support the implementation of the *Piano di studio* [4] and are therefore focused on the activation of the competence aspects (which are also present in the form of icons in the materials for teachers and in the materials for learners and with which an assessment for learning can be triggered) envisaged for mathematics (*knowing and recognizing*, *executing and applying*, *exploring and trying*, *mathematizing and modelling*, *interpreting and reflecting on results*, *communicating and arguing*, Figures 5 and 6), on the development of the competence goals prescribed for the end of second grade and fifth grade, and those related to transversal competences.

Figure 5. The six competence aspects envisaged for mathematics and identified by icons (original version in Italian).



(**a**) (**b**)

Figure 6. Competence aspects (here circled in red) in (**a**) a worksheet and (**b**) a didactical practice (original version in Italian).

- *Relational view of mathematics*: MaMa materials are permeated by a relational view of mathematics, rather than an instrumental one [29]: there is a specific focus on the cognitive processes at play, in particular on exploration, mathematization and modelling, communication, argumentation and interpretation of answers, and the explication and comparison of strategies rather than on the product itself (e.g., there is a stronger focus on problem contexts and estimation rather than on calculation for its own sake).
- *Flexibility*: all materials designed for use with learners (worksheets, attachments of didactical practices and game, supports) are editable in text and illustrations, so that for customization and differentiation.
- *Accessibility*: all materials are freely accessible and downloadable from the online platform.

3.1.3. The MaMa Platform and Its Dissemination

As mentioned above, all the materials are uploaded on an open access online platform (mama.edu.ti.ch), which has been specially designed by a web project manager, who interviewed numerous teachers in order to make it usable and intuitive even for those who feel less computer literate. On the platform, there are several filters common to the various materials (i.e., class, mathematical area, topics, and keywords) and others more specific to the type of material (e.g., worksheets can also be filtered by competence aspect and didactical practices also by competence goal), through which the teacher can refine his

or her searches. For each document, other related materials recalling similar themes are provided as further links. Moreover, by accessing the reserved area of the platform, it is possible to save the chosen materials online, creating personal collections.

The launch of the platform took place in December 2021, with the publication of the materials from the "Numbers and computation" area for first and second grade, followed in August 2022 by those for third, fourth and fifth grade. As of December 2023, almost all the materials for the "Geometry" area from the first to the fifth grade have been published. At the moment when this pilot study is carried out, the materials for the last area envisaged by the primary school curriculum, i.e., "Magnitudes and measures", are in production and are scheduled to be published in part by December 2024 and completed by August 2025. From the launch of the platform to date, around 285,000 downloads have been made, a result that testifies to what extent the materials are circulating among teachers in Ticino and, more generally, among Italian-speaking teachers.

3.2. Research Questions

As shown in the previous section, MaMa materials are not presented as a textbook or a set of worksheets to be followed in a certain order, but as a database of didactical resources to be selected and modified by teachers in order to be flexible and effective in different classroom contexts. These didactical resources are thus specifically designed to integrate static and dynamic aspects, seeking those "new balances between *static* and *dynamic* resources, between *using* and *designing* teaching resources" highlighted by Trouche and colleagues [3] (p. 1).

Conceiving the MaMa project as a "didactical engineering for teacher education and development", the research questions guiding the present study are as follows:

- How are MaMa materials used in teachers' practices?
- Does the use of such materials have an impact on teachers' disciplinary and didactical competences?

4. Methods and Materials

The survey was conducted in November 2023 by means of a voluntary and anonymous online questionnaire aimed at the first part of the published material, i.e., "Numbers and computation" area.

The questionnaire administered consists of 17 questions of different types: multiple-choice, multiple-answer, open-ended, 4-point Likert satisfaction or agreement scales. Many questions also include a space for comments. The questions are divided into five thematic blocks (see Table 1 for details and the Supplementary Materials for the full questionnaire text):

1. Questions on MaMa materials availability and searching.
2. Questions on MaMa materials ways of use.
3. Questions on MaMa materials evaluation.
4. Questions on MaMa materials impact on teaching profession.
5. Closing questions to improve MaMa materials.

To answer the first research question, data from the first two thematic blocks of the questionnaire (materials availability and searching, and ways of use) will be analyzed in Sections 5.1 and 5.2. To answer the second research question, data from the fourth thematic block (materials impact on teaching profession) will be analyzed in Section 5.3.

Data analysis method consists of a descriptive analysis conducted mainly at a quantitative level, especially for multiple-choice questions and Likert-type scales, deepened through a qualitative analysis of the available answers to open-ended question for justifying. In particular, for the analysis of the open answers of the fourth block, three categories have been created starting from available data, trying to highlight whether the answer refers to a disciplinary, didactical, or to both disciplinary and didactical plan.

Table 1. Overview of the questionnaire questions.

Thematic Block	Investigated Theme	Number and Type of Question
1a. Materials availability	**1.1** Way of accessing materials.	1 multiple-choice question.
	1.2 Frequency of access to the platform.	1 multiple-choice question.
	1.3 Features of platform navigation: intuitiveness, effectiveness, simplicity, clarity	4 4-point Likert satisfaction scales, one for each characteristic +1 open-ended question for comments.
1b. Materials searching	**1.4** Ways of searching for materials on the platform.	1 multiple-answer question.
	1.5 Creating online collections.	1 multiple-choice yes/no question.
	1.5.1 (linked to the previous one) Criteria used for collections.	1 multiple-answer question.
2. Materials ways of use	**2.1** Ways of using materials in one's own practices.	10 multiple-choice yes/no questions +1 open-ended question for comments.
	2.2 Use of editability.	1 multiple-answer question +1 open-ended question for comments.
3. Materials evaluation	**3.1** Materials features from the teacher's point of view.	15 4-point Likert agreement scales, one for each characteristic +1 open-ended question for comments.
	3.2 Satisfaction with each type of material: guidelines, contexts of meaning, didactical practices, problems, games, supports, worksheets.	7 4-point Likert-type satisfaction scales, one for each type of material +7 open-ended questions for justifying, one for each type of material.
	3.3 Materials features from the learner's point of view.	8 4-point Likert-type agreement scales, one for each characteristic +1 open-ended question for comments.
4. Materials impact on profession	**4.1** Change in mathematical knowledge.	1 multiple-choice yes/no question +1 open-ended question for justifying.
	4.2 Change in beliefs about mathematics.	1 multiple-choice yes/no question +1 open-ended question for justifying.
	4.3 Change in teaching methods.	1 multiple-choice yes/no question +1 open-ended question for justifying.
5. Closing questions to improve materials	**5.1** Suggestions for improving materials.	1 open-ended question.
	5.2 Overall evaluation of materials.	1 satisfaction scale from 0 to 10.
	5.3 Further remarks and comments	1 open-ended question.

Sample

This pilot study has been carried out at an intermediate stage of the project (November 2023), i.e., when the published material only covered the "Numbers and computation" area. Moreover, it was aimed at primary school teachers in Ticino who were aware of the existence of such materials and were potential users.

The teachers to whom the questionnaire was sent had in fact participated in in-service teacher education courses in which MaMa materials were presented: the first course *MaMaestri in fiera* (MaMa-teachers at the fair), in which around 200 in-service teachers at the Bellinzona district were obliged to participate, was aimed at the "Numbers and computation" area with the support of MaMa materials; the second course *Formarsi per formare in matematica* (Training yourself to train in mathematics), aimed at teachers from kindergarten to lower secondary school, had among its participants 18 primary school teachers who had the opportunity to learn about MaMa materials.

These teachers were initially asked to participate in the questionnaire only if they had used it in class. Of the teachers in the two groups to whom the questionnaire was sent,

49 voluntarily answered the survey in the first three blocks of questions and 46 in the fourth and fifth blocks.

The sample surveyed is distributed almost equally across the age groups: 24–30 years old (20%), 31–40 years old (29%), 41–50 years old (27%), over 51 years old (20%). Moreover, it is predominantly a female sample (around 84% of participants are women), which is not surprising given the school level involved. Most of the teachers (around 43%) have been teaching for more than 15 years, while the remaining ones are evenly distributed in the lower bands of teaching years: 1–5 years (18%), 6–10 years (18%), 11–15 years (21%).

5. Results

5.1. MaMa Materials' Availability and Searching

5.1.1. MaMa Materials' Availability

To investigate materials' availability, three questions were asked:

In response to the first question "*How do you find MaMa materials?*", 73% of the teachers state that they exclusively access the platform, while the remaining 27% state that they access both via the platform and by exchanging with colleagues; none state that they obtain materials exclusively from colleagues. The results highlight that teachers access the platform to consult and choose which materials to consider, possibly exchanging experiences with colleagues; this is an important aspect in terms of *appropriation* [18].

To the second question "*Since the activation of the MaMa platform, how often do you access it?*", 39% of teachers say they access it on average two–three times a month, 29% once a week, 24% once a month, and the remaining 8% two–three times a week or even more. In general, the frequency of access is at least once a month, which is high enough to indicate a frequent and regular level of searching for material on the platform, especially considering that all material is downloadable, so there is no obligation to return to the platform.

A confirmation of the easiness of navigation of the platform can be found in the answers to the third question: "*To what extent, in your opinion, does the navigation of the MaMa platform reflect the following features: intuitive, effective, simple and clear?*". Combining the answers "quite" and "very" gives the following percentages: intuitive (100%), effective (98%), simple (94%), and clear (98%). The remaining percentages refer to the degree "little" or are empty, while "not at all" was never chosen. There was also only one open comment from a teacher stating: "I sometimes have a little trouble finding what I am looking for". This comment highlights some criticality in terms of the simplicity of navigating the materials on the platform, probably due to their vastness which can sometimes be disorientating for the teacher. The overall results highlight how having studied with a web project manager the way teachers usually navigate online didactical resources had positive effects on the navigability of the platform.

5.1.2. MaMa Materials Searching

With regard to the searching for the material, two questions were asked:

From the answers to the first question "*What search methods do you use on the platform?*", in which it was possible to choose several answers from the eight options proposed, it emerges that the most commonly used method is to filter by topic (chosen by 94% of teachers), followed by entering specific keywords (chosen by 82% of teachers), and filtering by year (chosen by 71% of teachers). Less frequently used, are searching among related materials (chosen by 29% of the teachers) or the filters referring to the *Piano di studio*: competence goals and aspects, each of which is used by 24% of the teachers. The latter result was predictable, given the vast number of materials. Searching according to competence goals or competence aspects, without first choosing other filters, would in fact lead to a wide range of materials, so using this type of filter may only be functional at a later stage. Furthermore, it should be considered that several materials aimed at teachers, which can be traced by topic, contain at the bottom of the document several references to the curriculum, allowing the teacher to retrieve this information in any case. Teachers

therefore seem to search for MaMa materials in a targeted and conscious manner (only one teacher claims to search for materials without selecting filters).

These results highlight that many teachers search for materials on the platform according to criteria that they explicitly declare through filters. The design work is therefore supported by elements that can orient and help the searching. This highlights how MaMa materials are resources that "speak to teachers" [23] (p. 232) and how they are able to support "teachers in understanding the complexity of decision-making about classroom tasks [. . .] often regarded as influencing, and sometimes determining, the degree of students' opportunities to learn" [19] (p. 106).

The answers to the second question "*Do you create online collections of MaMa materials?*" also show that the creation of personal online collections, exploited by 33% of the teachers, takes place according to criteria that are once again by mathematical topic (chosen by 88% of the teachers who create collections), by class (38%), and in four cases the type of material or the competence goal of the curriculum is also indicated along with the abovementioned criteria.

5.2. MaMa Materials Ways of Use

5.2.1. Use of MaMa Materials in Teaching Practice

In order to investigate how the materials are used, the following question was asked: "*For each of the following options, please put a 'Yes' or 'No' according to how you use MaMa materials from the "Numbers and computation" area in your teaching practice*", which was followed by ten statements plus space for any open-ended comments.

As can be seen from the results (Figure 7), all the teachers who responded to the questionnaire state that they integrate MaMa materials with other materials of their own creation or from other sources. This confirms that those who use these materials do not adopt them *tout court* but use them to support their own critically and consciously thought-out instructional design, without relying on ready-made solutions [12].

A total of 94% of the teachers claim to select and use the materials in a personal order; only 4% claim to use the materials exclusively in the sequence proposed on the platform. However, it should be considered that there are more "gateways" to the materials and more possible ways to obtain a sequence of materials after setting possible initial filters, which also are different; for example, different learning paths can be identified via the sequence of the selected types of materials or through the suggested links to related materials.

A total of 92% of the teachers confirm that they use MaMa materials not only in the design, but also in the classroom implementation of their own learning paths; this means that they do not only rely on the materials intended for teachers, which are useful for guiding the design, but also use those intended for learners. Our interest will also be in understanding how these latter materials are used, in particular whether teachers modify them and for what purposes (see Section 5.2.2).

A total of 84% of the teachers also state that they read the guidelines, which deal with the disciplinary and didactical content of the various topics involved, in order to check the mathematical content that will then be explained in the classroom. In addition to the guidelines, 63% of the teachers declare that they personally use MaMa materials to better understand the content to be explained to pupils for feeling more confident. A need seems to arise, teachers want to personally educate themselves both from a disciplinary and didactical point of view before proposing materials to their pupils. These results highlight how the materials aimed at teachers are highly regarded; MaMa does not reduce itself to a simple set of worksheets aimed at pupils. At the same time, it emerges how MaMa materials come to affect the "First space" [15], i.e., the teachers' academic knowledge. This can possibly be confirmed by the impact that teachers think the materials have on their expertise (see Section 5.3).

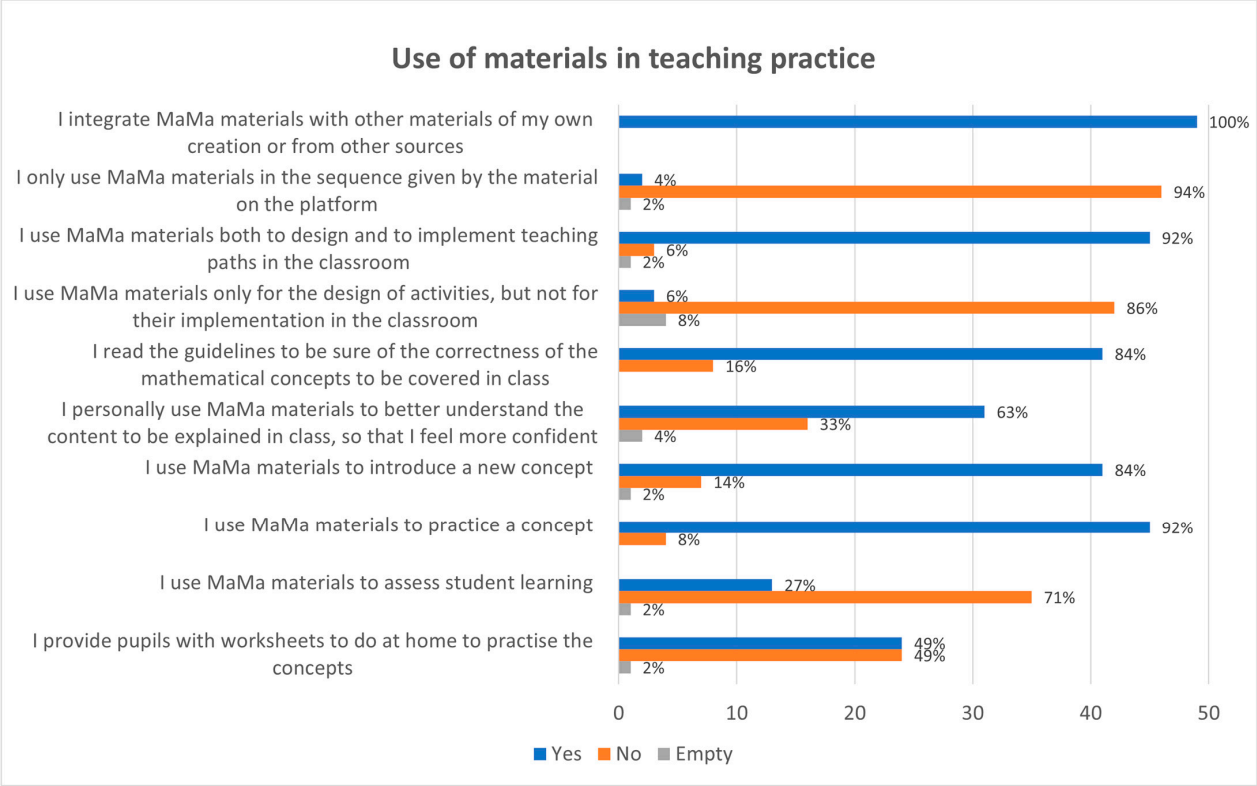


Figure 7. Responses to the question on MaMa materials use in one's own teaching practice.

In terms of purpose of use, many teachers state that they use MaMa materials in the classroom to introduce a new concept (84%) and to practice a concept (92%), while only 49% provide pupils with worksheets to do at home to practice concepts. It should be noticed that the school in the Canton of Ticino is full time for everyone, and few requests are expected to be carried out autonomously at home. Moreover, the MaMa materials are designed to compare with others, in particular with one's peers and the teacher.

A smaller percentage of teachers (27%) use materials to assess pupils' learning. These results may have been affected by the fact that no specific materials aimed at assessment have been created in MaMa and no meetings or teacher education courses have been held yet about how materials can be used from this point of view. Another element that certainly affected the results, and which is not investigated in this pilot study, is the teachers' idea of assessment. The *Piano di studio* promotes formative assessment, designed for learning, and the MaMa materials are suitable to implement this type of assessment, thanks to their focus on the competence aspects at stake and the possibility of discussing them with the learners, in order to focus on metacognition and thus make them increasingly responsible for their learning [30]. In addition, in MaMa, multiple opportunities are offered whereby the learner is invited to discuss with one or more peers supporting peer assessment [30]. We do not know, however, whether the 27% of the respondents who stated that they use MaMa materials to assess learning use it in a formative sense, or rather in a summative one. Nevertheless, assessment is certainly an area in which teachers' beliefs can be further investigated, since it also partly affects the beliefs that teachers have about mathematics and the way mathematics is taught (see Section 5.3). Beliefs which, as we noted in Section 2, can be very influential on the way teachers select and use didactical resources [24,25].

It should also be noted that the use of such didactical resources with a view oriented by formative assessment requires a deep *appropriation* work by the teacher, precisely because it presupposes integrating and adapting them to the competences of individual learners or a group of learners.

5.2.2. Use of Editability

With regard to editability which, as anticipated, involves a deep level of *appropriation* of the materials, four options of interaction with the resources were provided and teachers were asked to choose the one(s) that best represented their own habits, possibly adding an open-ended comment: "*Of the following options, choose the ones that best represent the use of editability of MaMa materials in your teaching practice*".

The answers to this question show that 88% of the teachers state that they use editability. The most popular editability option (chosen in 72% of cases) is that of making several versions of each worksheet so that they are calibrated to the skills of each group of learners, thus creating an individualized differentiation. This is followed by the option of editing the attachments of games and didactical practices (chosen in 58% of cases), even though several more or less complex variants of such materials are often already provided and represent a useful basis to be used according to needs. Finally, a further option, though less used (33% of cases), is that of editing the worksheets by changing texts/numbers, creating one type of worksheet for all pupils but adapted to the class context, thus creating an a priori differentiation. From these results, it emerges that teachers have activated a process of *appropriation* regarding MaMa materials, within the "design as Third space" [14], since, through differentiation, they relate their own academic knowledge with that observed in the learners.

However, there are 12% of teachers who state that they do not use the editability of materials, probably also due to possible technical difficulties, as some open comments point out (e.g., "However, this option creates some difficulties for me at Font level (it changes it for me). As I do not appreciate the editing, I rarely do it" (This teacher probably did not download the free program recommended on the platform – Adobe Acrobat Reader – that complies with the editability of the materials)). These teachers *integrate* MaMa materials into their own practice, as they are, without adapting them to their own class or individual learners; a practice usually applied by teachers with traditional textbooks, when the materials are not editable [20].

5.3. MaMa Materials Impact on Teaching Profession

In order to answer the second research question, we analyze the answers to the fourth thematic block of the questionnaire consisting of three questions, which were answered with "Yes" or "No", followed by a space to justify the answer. We decided to ask these questions at this intermediate stage of the project to capture the teachers' beliefs, well aware that a significant impact on the teaching profession takes a long time [1,3]. In the following, we report the results obtained from each of the three requests.

5.3.1. Change in Mathematical Knowledge

To the first question "*Do you think the MaMa materials have changed your mathematical knowledge?*", 52% of the teachers state that the MaMa materials have had an impact on their disciplinary knowledge. Of these, 63% give a justification via open commentary, which we have summarized in the following three categories: disciplinary, didactical, or mixed.

- *Disciplinary*. In 40% of the justifications, the change in mathematical knowledge is related to content aspects: "I have learnt a lot about concepts that were not very clear to me", "They help me to put my knowledge in order where I feel the need". This is also due to the greater amount of time teachers feel they have available to prepare, thanks to the material provided: "The excellent material encourages me to inform myself more and prepare more. The fact that materials are already ready leaves me time to document myself, time that was previously mainly used to search for material and create worksheets from the beginning".
- *Didactical*. In 13% of the justifications, changes in mathematical knowledge are more attributable to the didactical plan: "More than changing my knowledge, they have brought my attention back to new and more stimulating ways and approaches".

- *Mixed*. A total of 47% of the justifications for change in mathematical knowledge refer to both levels, disciplinary and didactical, as can be seen from the following examples: "Knowing that I can draw on such rich and correct material acts as a stimulus to revise old didactical itineraries and make them more appealing and more correct from a mathematical point of view", "The MaMa worksheets have enabled me to learn about different ways of approaching topics and have allowed me to 'brush up' on certain concepts", "The MaMa materials have encouraged me to go deeper into the various topics related to mathematics. These materials also stimulate pupils proposing activities that are close to their reality while at the same time prompting them to reflect on the strategies adopted and not just apply them mechanically".

A total of 48% of the teachers state that they have not changed their mathematical knowledge with the use of MaMa materials. Of these, 23% give explanations that also have a positive connotation: one highlights not a change in knowledge but a greater precision in mathematical language ("Not changed, but sometimes materials have given more precision in terms"), other two highlight an alignment of the MaMa materials with their previous mathematical and didactical knowledge (e.g., "The topics in the MaMa materials do not present significant changes in mathematical knowledge compared to other approaches or ways of teaching mathematics that I have adopted in the past"). The others observe that the approach of the MaMa materials traces teacher education courses that are widespread in the Canton of Ticino. This can be interpreted as an indication of the fact that in a small area such as the Canton of Ticino, a certain resonance can be felt between training and materials for teacher education and development, as well as a closeness between institutions and schools that favors consistency in the proposals. The MaMa materials are thus perceived by the teacher–users as materials for their education and professional development, with a greater impact at the disciplinary level, with implications at the didactical level as well. This is an important result in terms of *transformation* [19], since teachers use MaMa materials both in the design phase (for a "more confident" and "varied" work, as made explicit in some comments) and above all for a disciplinary consolidation of their own knowledge.

5.3.2. Change in Beliefs about Mathematics

To the second question "*Do you think that MaMa materials have changed your beliefs about mathematics?*", 80% of the teachers state that there is no change in their beliefs. Their justifications (given in 27% of cases) point to a view of mathematics which is already in line with MaMa one (see Section 3): "I have always liked mathematics and I think MaMa confirms how interesting and fun it can be, as well as being all around us", "MaMa is simply a continuation and concrete resumption of what I had already learnt".

On the other hand, 17% of the teachers declare a change in their beliefs about mathematics, and three explanations are given, one of which refers to the disciplinary level ("Materials for teachers have given me a clear and precise line. Previously, the search for information led me to different variants and points of view that sometimes confused me"), one to the didactical level ("MaMa has helped me to propose more attractive activities that 'make the pupils think' about many aspects"), and one directly to beliefs ("Mathematics seems even more interesting to me as it is applicable to everyday life").

It has to be considered that a change of beliefs about mathematics requires a lot of time and the use of MaMa materials also in the other mathematical areas, "Geometry" and "Magnitudes and measures". Therefore, the process has just begun.

5.3.3. Change in Teaching Methods

To the third question "*Do you think that MaMa materials have changed the way you teach mathematics?*", the teachers answered similarly to the previous question.

74% of the teachers do not notice any changes in their teaching methods brought about by the use of MaMa materials, and 41% of them provide an explanation, mainly testifying to the fact that they already work in line with the teaching methods promoted by MaMa (e.g., "No, because I am used to working in the different topics with games, workshops

and real problem situations"). That being said, almost half of the explanations nevertheless emphasize small variations or enrichments made possible by the use of MaMa materials (e.g., "I was already used to working in similar ways, but the materials allow me to do so more frequently and to add new and interesting moments", "MaMa definitely manages to enrich these moments").

Instead, 26% of the teachers declare a change in their teaching methods. Half of them give a reason for such a change, linking it to new teaching approaches, including workshop activities (e.g., "Following the proposals I automatically found myself planning more group work or workshops"), games (e.g., "It pushes me to dare, integrating more games and more practical situations"), and varied learning moments (e.g., "They give me more cues for different learning moments"). One of the explanations specifically refers to a deeper pupils' reflection "on the competences and processes at play", suggesting a more confident implementation of the curriculum based on the various competence aspects.

This analysis highlights the fact that MaMa materials resonate with teachers' practices and in some cases reinforce them, pushing them "to dare" more with the implementation of certain teaching methods promoted by the curriculum. This aspect is to be considered very positive, also because, as stated in Section 3, an attempt was made to create the materials starting from "good practices" in the area.

6. Discussion and Conclusions

In this article, we have analyzed the interaction of primary school teachers with the didactical resources aimed at mathematics teaching and learning produced and disseminated by the *MaMa—Matematica per la scuola elementare* (MaMa—Mathematics for primary school) project. These materials are proposed as a "second-generation didactical engineering", i.e., aimed at teacher education and professional development with a strong impact on pupils. In particular, we asked how MaMa materials are used in teachers' practices and whether they believe that the use of these materials has an impact on their disciplinary and didactical competences.

Concerning the first question, a very conscious and flexible use of MaMa materials by the teachers involved in this pilot study was noted, as regards both the design and the implementation of didactical practices, with a particular attention to the needs and specificities of the learners. This has been highlighted by the variety of practices activated and the differentiation of the materials according to the different needs of the context. In fact, reading the results in terms of *adoption*, *integration*, *appropriation*, and *transformation* of resources [19], it emerges that MaMa materials are *integrated* into the practices of teachers, who select them themselves, using the platform filters, in a targeted and conscious manner and use them in dialogue with resources from other sources or of their own creation. Not only this, but they also *appropriate* MaMa materials since most of the teachers who use them exploit their editable nature, modifying them and using them as materials that can be adapted to the needs of the class.

Regarding the second research question, MaMa materials seem to have an impact on teachers' competences, being perceived by those who use them as resources for their own education, especially on a disciplinary level, with implications on the didactical level as well. Such materials are also seen as a design support, in line with practices already in use by teachers, which enrich them and allows them to "dare" more.

MaMa materials thus seem to support "instructional design as a Third space" [11], i.e., the work at the intersection between teachers' knowledge, which is reinforced and consolidated by their use, and learners' knowledge, which is considered and enhanced through the editable nature of the materials.

In analyzing and discussing teachers' interaction with these didactical resources; however, we are well aware of certain aspects.

The first aspect is the complexity of the vast system of resources that teachers have to deal with on a daily basis, an aspect that creates difficulties and sometimes disorientation in searching (e.g., [2,3]). In this regard, Chalambrous and Hill [25] highlight the challenges

inherent in developing curriculum materials without trespassing into the production of voluminous materials that would be difficult to use. The MaMa project tries to maintain the balance between a mostly intuitive, effective, simple, and clear navigation and a wide range of meaningful and educationally validated materials. Confirmation of this balance comes from the majority of teachers' responses, stating that they find "a clear and precise line" in these resources. This "defined point of view" is often recognized as being in line not only with the curricular indications to be implemented in the classrooms, but also with teachers' own practices and beliefs about mathematics. In order to achieve this balance and this recognition, which are fundamental in the *appropriation* of resources by the teacher–user, the MaMa project has focused on the proximity between the world of research and that of the school, including in the team figures with different profiles and proposing "good practices" tested over the years in the region.

The second aspect is the temporal dimension, whose fundamental role is recognized in the process of resource *appropriation* by all research on the subject, particularly in Gueudet and Trouche [1] who state the following: "[Time] is an important matter. The resources evolve, are modified, combined; documents develop according to the processes of genesis and bear new resources... Long-term evolutions, but also more limited events, must be taken into account" (p. 207). The process of *appropriation* takes time not only in the direction in which the teacher transforms a resource, but especially in the other direction, which involves a *transformation* of the teacher's beliefs through the use of the resource itself [19]. The fact that the teachers participating in this study were only able to get to know and experience the MaMa materials for a limited time (two years for the "Numbers and computation" materials in first and second grade, only one year for those in third, fourth, and fifth grade), and that the published materials do not yet involve all areas of mathematics, is certainly an element to be taken into account when interpreting the data collected, perhaps also behind the small number of responses obtained to the questionnaire.

It should be mentioned in this regard that the work presented in this article is an orientation pilot study, which was purposely conducted at an intermediate stage of the project, in order to guide the second part of the project. In order for the *transformation* process through the use of the MaMa materials to lead teachers to embrace the principles of the materials in a profound and conscious manner, it is necessary to have the completeness of the materials, specific teacher education courses, and more time for implementation and reflection.

The sample of the study, which is currently numerically small and selected from those who were actually introduced to the materials through courses, will certainly need to be expanded when all the materials will be on the platform. The improved questionnaire should be sent to all teachers in the region in order to understand also how many of them use the materials, expanding the data collected for those who use them and deepening the motivations of those who do not use them.

This is because the results collected from this pilot questionnaire already reveal interesting considerations on the use and impact of these materials in teaching practices that could be further investigated and expanded upon. To do this, a survey carried out with a questionnaire at an intermediate stage is only the first step, to guide future choices in the production and dissemination of the materials. It will also be interesting to supplement these evaluations with the observation of some didactical paths realized in the classroom with these materials, in order to follow the actual *transformation* process in the design and realization phase. In addition, the survey could be supplemented by individual interviews to deepen the transformation process of the resources and its impact on the profession.

Once the project is completed, these further elements will make it possible to confirm the potential, which emerged in an embryonic stage in this study, of the MaMa project as a "didactical engineering for teacher education and development" [12], and to observe its indirect effects on the pupils' learning.

Supplementary Materials: The full text of the questionnaire as it was administered online is available at the following link: https://drive.switch.ch/index.php/s/9Gfcm5mtJT17xuH (accessed on 18 April 2024).

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Informed Consent Statement: Informed consent was obtained from all subjects involved in the study. Informed consent I s present in the introduction of the questionnaire, which is available in the Supplementary Materials. Participants needed to click on "I agree" to proceed to the questions.

Data Availability Statement: Data are stored confidentially by the authors but are not made available at the moment, as this is a pilot study. Nevertheless, an organization of data availability will be arranged for the continuation of the study, which includes a revision and readministration of the questionnaire to a wider sample.

Conflicts of Interest: The authors declare no conflict of interest.

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