2.5.2. Modified Mann-Kendall trend test and Sen's slope estimator.

The MK test statistic S is given by equation 3.

$$S = \sum_{b=1}^{n-1} \sum_{a=b+1} \sin(x_a - x_b) \tag{1}$$

Where x_a is the order data value, n is the length of observation. The value of the sign of the test statistic is computed as shown in equation 4.

$$\sin(x_a - x_b) = \begin{cases} 1 & if\ x_a - x_b > 0 \\ 0 & if x_a - x_b = 0 \\ -1 & if x_a - x_b < 0 \end{cases} \tag{2}$$

For $n \geq 10$, the S statistic is nearly normally distributed with mean zero {E(S) = 0} and variance (V) as in equation 5.

$$V(S) = \frac{n(n-1)(2n+5) - \sum_{b=1}^{nb} c_b(b)\,(b-1)(2b+5)}{18} \tag{3}$$

c_b = To the extent b, the number of ties or duplication, the total number of ties in the dataset is nb. If the $n \geq 10$, then the normalized test statistic Z_s is given by equation 6.

$$Z_s = \begin{cases} \frac{S-1}{\sqrt{V(S)}}, & (if\ S > 0) \\ 0, & (if\ S > 0) \\ \frac{S+1}{\sqrt{V(S)}}, & (if\ S > 0) \end{cases} \tag{4}$$

Positive (negative) value of Z_s indicates an increasing (decreasing) trend. The value of α was kept as 0.05 for analysis. The null hypothesis that no significant trend exists in the dataset is rejected if $|Z_S| > 1.96$.

However, autocorrelated time series datasets need to be corrected for autocorrelation (Hamed & Rao, 1998) between rank of the observations ρ_k by subtracting Sen's median slope from the slope of the data. The correction factor is computed by equation 7, where n and n_s^* are the actual and effective number of observations respectively.

$$\frac{n}{n_s^*} = 1 + \frac{2}{n(n-1)(n-2)} \times \sum_{k=1}^{n-1} (n-k)(n-k-1)(n-k-2)\rho_k \tag{5}$$

The corrected variance is then calculated by equation 8 as given.

$$V^*(S) = V(S) \times \frac{n}{n_s^*} \tag{6}$$

2.5.3. Sen's slope estimator

A non-parametric approach to estimate the magnitude of the trend in a sample of N pairs of data formulated by Sen was used in the study (Sen, 1968).

$$Q_i = \frac{x_j - x_k}{j - k} \text{ for } i = 1, 2, \ldots, N \qquad 7$$

where x_j and x_k are data points at times j and k (j>k), respectively.

The N values of Q_i are ranked from smallest to highest to compute the median of slope (Sen's slope) as

$$Q_{med} = \begin{cases} Q_{|(N+1)/2|}, & (if\ N\ is\ odd) \\ \frac{Q_{|N/2|} + Q_{|(N+2)/2|}}{2}, & (if\ N\ is\ even) \end{cases} \qquad 8$$