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On the Importance of Future, Precise, X-ray Measurements in Kaonic Atoms

Sławomir Wycech [1] and Kristian Piscicchia [2,3,*]

[1] National Centre for Nuclear Research, Andrzeja Sołtana 7/3, 05-400 Otwock, Poland; slawomir.wycech@ncbj.gov.pl
[2] Centro Ricerche Enrico Fermi—Museo Storico della Fisica e Centro Studi e Ricerche "Enrico Fermi", Via Panisperna, 89a, 00184 Roma, Italy
[3] Laboratori Nazionali di Frascati (INFN), Via Enrico Fermi, 54, 00044 Frascati, Italy
* Correspondence: kristian.piscicchia@cref.it

Abstract: Progress in the construction of precise X-ray detectors allows measurements of energies and widths of "upper levels" in K^- mesic atoms. These can be used to determine sub-threshold Kaon-nucleon amplitudes, which are important in investigations of nuclear states of these mesons. The special case of the 2P state in Kaonic Helium is discussed and used to check the properties of the K^- proton quasi-bound state. Similar attempts in other elements indicate a need for new, precise measurements.

Keywords: kaonic atoms; X-ray measurements; kaonic nuclear states

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1. Introduction

Studies of K^- mesonic atoms started in the 1960s with the experiment of Wiegand [1]. Soon, it turned out that atomic level shifts due to nuclear interactions of K^- meson are repulsive, while scattering experiments indicated a strong K^- proton attraction. The experimental investigation of kaonic hydrogen [2–4] put in evidence an attractive-type strong interaction at a threshold between the kaon and the proton, but still in contradiction with the results of low-energy scattering experiments. This so-called "kaonic hydrogen puzzle", together with the measurement of the shift of the transition to the 2P level in helium, much larger than the theoretical prediction, lead to the development of refined experiments. The technological developments in accelerator physics lead to the construction of the DAΦNE collider at LNF-INFN in Italy, and of kaon extracted beams in Japan, first at KEK and then at J-PARC. DAΦNE delivers low-momentum (about 127 MeV/c), almost monochromatic kaon beams, generated by the decay of ϕ-resonance, which is produced in electron-positron collisions. KEK and J-PARC delivered high-intensity, high-energy kaon beams which are degraded to be stopped in the targets. In parallel, the progress in the development of solid-state X-ray detectors (Silicon (Lithium) detectors, Charge Coupled Devices and, more recently, fast Silicon Drift Detectors with timing capabilities) allowed progressive improvement of the detection of a faint kaonic atom signal over the huge accelerator background. Moreover, lightweight gaseous high-density cryogenic targets were built, aimed to avoid the Stark effect. The aforementioned technological progresses allowed the performance of new experiments (see, e.g., [5–7]) which solved the the kaonic hydrogen puzzle and measured kaonic atom transitions in a series of low-Z targets, contributing to the understanding of the atomic cascade processes in kaonic atoms. Extremely difficult was the measurement of kaonic deuterium due to the small expected yield of the last transition to the fundamental level and the broadness of the transition line. High-performance Silicon Drift Detectors were developed and new cryogenic techniques were applied to the realization of a high-density cryogenic gaseous deuterium target, which are exploited in the ongoing SIDDHARTA-2 experiment [8].

First observations in nuclear emulsions indicated that K meson is captured via Auger electron emission. Hence, the initial atomic states of mesons have radii close to the parent electronic atom radius and correspond to high n values. Subsequent X-ray measurements found that radiative cascades terminate by nuclear captures of mesons from states of high angular momenta. Due to the centrifugal barriers, nuclear interactions are localized at nuclear surfaces. These findings offered a unique chance to study the nuclear structure in this region. Two questions evoked particular interest: first the possible existence of a neutron excess at large nuclear radii (called neutron skin or neutron halo), and second the existence of strong α-particle-type correlations at nuclear surfaces. These nuclear properties were studied by means of the detection of capture products: π mesons and hyperons. Soon, it turned out that understanding of the interactions involved is not easy due to attractive K^-–p interactions. This attraction was attributed to the existence of $\Lambda(1405)$, conceived as a K^-p quasi-bound state, decaying predominantly into Σ, π pairs [9]. This idea led to the expectation that such a mechanism may lead to the formation of nuclear quasi-bound states of K^- mesons but also strongly complicate its description.

With the advent of quark physics, it was found that $\Lambda(1405)$ has no simple or natural location in quark symmetry schemes. Other approaches were undertaken, and we discuss some models in this text. All these models are constrained to reproduce elastic and inelastic K^- nucleon interactions above the K^-p threshold. On the other hand, kaonic-atom studies and the search of bound Kaon-nucleus systems require knowledge of the interaction below the K^-p threshold. Experimental tests of the related scattering amplitudes in nuclear systems are complicated by multiple scattering and final state interactions, in the case of inelastic processes.

This work discusses possibilities to study the structure of $\Lambda(1405)$ with measurements of selected X-ray transitions in Kaonic atoms. Of interest are transitions to so-called "upper levels" characterized by extremely small atomic nuclear overlap. In such systems, the atomic levels are essentially determined by the interactions of the meson with a single nucleon, and the levels shift due to nuclear interactions and level widths due to nuclear captures are directly related to Kaon-nucleon scattering amplitudes. Experimental investigations of this type have been made possible by new, very precise detectors, and the perspectives are promising.

It turns out that the 2P atomic state in light K mesic atoms is an ideal object for performing such a test. Because of the centrifugal barrier, the meson just grazes the nuclear surface and interacts only with valence nucleons. In He nuclei, such nucleons are bound by some 20 MeV, and this localizes the energy in the KN center of mass system in a narrow region around 34 MeV below the threshold.

The extrapolation goes quite far, even below the center of the $\Lambda(1405)$ spectrum, and makes it possible to check sizable differences between the models. Another advantage is that 2P level widths and shifts may be calculated essentially in the leading order of multiple scattering expansion. Now, the greatest advantage is that X-ray measurements reached consistency with K^- He scattering at low energies. It took some 50 years of hard experimental work to reach this result, which is also analyzed in this work.

The formalism of the analysis is presented in Section 2. In Section 3, measurements of the 2P atomic state in the K mesic He atom are discussed, several theoretical results are checked against data, and some new measurements are suggested.

2. Mehtod of Analysis

Pioneering X-ray measurements in He were performed by Michael [10]. Later, Wiegand [11] observed the 3D $\rightarrow$ 2P transitions and extracted the shift of the 2P state. The first measurements of line width came from the Rutherford Appleton Laboratory (Chilton, UK) [12,13]. These created a "K-α puzzle" as the large level shift indicated a quasi-bound K-α P wave state that could not be generated by a standard $\Lambda(1405)$ effect, and required a new scenario that has never been found. Contemporary experiments from RIKEN and J-PARK [14] bring agreement with the old K-α scattering result of Mazur, which, apparently, until the recent years, was viewed with some skepticism. All numbers are presented in Table 1. In this work, we discuss the impact of the scattering results. We also indicate possibilities of other X-ray experiments which have some advantages over scattering experiments and where the studies of "upper levels" have good chance of success.

Table 1. The first six lines of the table show the results obtained by X-ray measurements for the 2P Kaonic ^{4}He shift and width in eV. Errors in brackets represent the sum in quadrature of the statistical and systematic uncertainties. The last two lines offer two best-fit results from the scattering experiment.

Reference	Level Shift	Level Width
Wiegand [11]	40 (30)	
Batty [12]	35 (12)	30 (30)
Baird [13]		100 (30)
Okada [14]	$2 \pm 2 \pm 2$	
Bazzi [15]	$-2 \pm 2 \pm 5$	$14 \pm 8 \pm 5$
Sirghi [16]	$0.2 \pm 2.5 \pm 2$	8 (10)
Mazur [17]	−0.09 (0.17)	0.94 (0.19)
Mazur [17]	−0.16 (0.31)	0.84 (0.18)

2.1. Relation between Level Shifts and Scattering Amplitudes

Detection of X-rays emitted from hadronic atoms provides energy levels. These are shifted in comparison to the electromagnetic levels by ΔE due to nuclear interactions. Level widths Γ, due to meson annihilation, are also provided in this way. For a given main atomic quantum number, n, and angular momentum L, these complex level shifts may be related to the corresponding hadron-nucleus L-wave scattering parameter A_L (length for $L = 0$, volume for $L = 1$). The relation known as Deser–Trueman formula [18,19] is obtained by expansion of ΔE in A_L/B^L, where B is the Bohr radius. The scattering amplitudes are measured experimentally and analyzed in terms of partial waves which determine A_L. The latter involve inner Coulomb corrections. For S waves, this relation is

$$\Delta E_{nS} - i\Gamma_{nS}/2 = E_{nS} - \epsilon_{nS} = \frac{2\pi}{M_r}|\psi_n(0)|^2 A_0(1 + \lambda_0 A_0/B). \tag{1}$$

Here, $\psi_n(0)$ is the atomic wave function at the origin and M_r is the meson–nucleus reduced mass. Formula (1) is accurate to the second order in A_0/B; higher terms in this expansion are not needed for small Z nuclei. This is obtained in the non-relativistic limit. Changes are to be introduced into electromagnetic energy ϵ_{nS} which is composed of Bohr's atom energy, $\epsilon_n^0 = -M_r(Z\alpha)^2/2n^2$, corrected for relativity, radiative effects and nuclear polarization, α being the fine structure constant and n the main atomic quantum number. λ is a coefficient in the expansion of powers in (A/B).

The Bohr radius is given by $1/(Z\alpha M_r)$. In $1S$ states, one has $\lambda_0 = 3.154$, and with $A_0 \approx 1$ fm, the second-order term in Equation (1) constitutes a few-percent correction. Such corrections are negligible in higher angular momentum states, and for these states, a simpler relation,

$$\Delta E_{nL} - i\Gamma_{nL}/2 = \epsilon_n^o \frac{4}{n} \Pi_{i=1}^L \left(\frac{1}{i^2} - \frac{1}{n^2}\right) A_L/B^{2L+1} (1 + \lambda_L A_L/B^{2L+1}), \tag{2}$$

is due to Lambert [20]. The second-order correction and examples of inner Coulomb correction may be found in ref. [21]. For $2P$ levels, $\lambda_1 = 1.866$. Thus, measurements of level shifts are equivalent to measurements of parameters involved in the scattering amplitudes, f, which, in the low energy expansion, are given in terms of initial $\boldsymbol{k}$ and final $\boldsymbol{k'}$ c.m. momenta by

$$f(\boldsymbol{k}, E, \boldsymbol{k'}) = A_0(E) + 3\,\boldsymbol{k} \cdot \boldsymbol{k'}\, A_1(E). \tag{3}$$

As experiments with strange particles offer rather low statistics, the agreement of X-ray experiments and scattering experiments offer reliable results and yield valuable data to test theories.

At this point, we remind that A_L which enter Relations (1) and (2) are parameters due to all short-range interactions and contain also the inner Coulomb corrections. With the procedure used here, the Coulomb field is due to the target nucleus and Coulomb effects are included in atomic wave functions. However, the basic scattering lengths and volumes, a_0 and a_1, for $\bar{p}$-N systems are to be calculated with inner Coulomb corrections. The scattering lengths are defined with a negative absorptive part. Thus, $A_L = Re\ A_L - i\,|Im\ A_L|$, and a bound state close to threshold is signaled by a *large and positive* $Re\ A_0$.

2.2. Relation between Level Shifts and Sub-Threshold K^- N Scattering Amplitudes

The meson, bound into an atomic orbit, scatters on a nucleon, bound in a nucleus. We calculate atomic levels in a quasi-three body system consisting of meson K(1), nucleon N(2) and residual nucleus R(3). Such approach makes sense only for peripheral interactions, and we limit ourselves to such cases. The purpose is to extract the energy dependence of elementary K^- N scattering amplitudes. The relevant energy involved in the c.m. of this pair is given by three terms:

$$E_{cm} = -E_s - E_a - E_r, \tag{4}$$

nuclear binding of nucleon E_s, atomic binding of meson E_a and E_r, a recoil energy of the K^- N pair with respect to the residual nuclear nucleus. The latter is given by $E_r = P^2/2\mu$, where μ is the reduced mass in the (KN)-R system. E_r reflects the distribution of total K^- N pair momentum P which has to be calculated with the use of kaonic and nucleonic wave functions. It offers some spread of c.m. energies involved in K^- N collisions. Details of calculations performed in the quasi-thee body system are given in ref. [22]. Some values of the average sub-threshold energies and the spread due to recoil are indicated in Table 2 for the lowest Z nuclei. All terms are negative and locate the energies below the K^- N threshold.

Table 2. Sub-threshold energies E_{cm} in K^- N subsystems in lightest atoms, averaged over separation and recoil energies. The numbers in parentheses are recoil energies, the latter indicating the spread of E_{cm} involved. MeV units, all numbers are accurate to 0.1 MeV.

Atom\State	**1*S***	**2*P***	**3*D***
$K^-\ {}^1H$	0	0	0
$K^-\ {}^2H$	−11.7 {5}	−7.9 {2}	−5.5 {1}
$K^-\ {}^4He$	−34 {10}	−33 {10}	−34 {10}

We compare imported sub-threshold S wave KN scattering amplitudes generated in a half-of shell form $a_0(E_{cm})$. Next, these are used to generate the level shifts in single and multiple KN scattering cases. To this end, we use two basic KN pseudo-potentials,

$$V(S) = \frac{2\pi}{m_r}\, a_0(E_{cm})\, \delta(\mathbf{r_{12}}); \;\; V(P) = 2\,\frac{2\pi}{m_r}\, \overleftarrow{\nabla} a_1(E_{cm})\delta(\mathbf{r_{12}})\overrightarrow{\nabla}, \tag{5}$$

where m_r is the reduced mass of the KN pair and $a_0(E_{cm})$ is the S wave scattering length at c.m. energy E_{cm}. Kaon nucleon systems may exist in two isospin states $I = 0, 1$: the K^-n pair makes a pure I = 1 state and K^-p makes an equal mixture of $I = 0$ and $I = 1$ components. Interactions in $I = 0$ contain coupling to the elusive $\Lambda(1405)$ particle. Table 3 shows that it dominates the atomic level shifts and widths. The purpose of upper level studies is to extract $a_0(E)$ and in particularly the part due to $\Lambda(1405)$. Interactions in the $I = 1$ state provide a background. Another source of background is the $V(P)$ interaction dominated by isospin $I = 1, J = 3/2$ unstable particle $\Sigma(1385)$ decaying mostly to the Λ, π channel. This state contributes well-known resonant amplitude

$$a_1 = \frac{\gamma_{KN}\gamma_{KN}}{E_{cm} - E_\Sigma + i\Gamma/2}. \tag{6}$$

Table 3. The S wave amplitudes in the region of -34 ± 6.6 MeV obtained in several theoretical approaches, (fm) units.

Model	K^-p	K^-n
P [23]	$-1.15 - 2.52i$	$-0.67 - 0.37i$
KM [24]	$-1.05 - 1.53i$	$-0.52 - 0.33i$
$M1$ [25]	$-1.20 - 1.41i$	$-0.07 - 0.01i$
$M2$ [25]	$-0.49 - 0.70i$	$+0.09 + 0.13i$
$B2$ [26]	$+0.32 - 0.42i$	$+0.20 - 0.12i$
$B4$ [26]	$-0.05 - 0.35i$	$+0.70 - 0.02i$

$K^-n \to \Lambda(\Sigma)\pi^0$ reactions were studied experimentally [27]; they yield $E_\Sigma \simeq 1382$, decay width of 36 ($\pm$2) MeV and the $\Sigma\pi$ branching ratio of 0.13 ($\pm$0.01) [28]. For the coupling to the $\bar{\text{K}}$N channel, the SU(3) value $\gamma^2_{\bar{K}N}/\gamma^2_{\Lambda\pi} = 2/3$. This value is consistent with the experimental result of Brown, which yields 0.57 ($\pm$0.18) [29]. The levels in K-mesic Helium discussed here involve the KN energy of about 1400 MeV close to resonant energy E_Σ, and Formula (6) is satisfactory. However, for weakly bound nucleons, for example, in deuteron, it is advisable to extend the resonant description to energy dependent width and γ_{KN} as discussed in ref. [30].

To calculate the atomic level shift, let us begin with first-order perturbation for S wave interaction. We need atomic wave function ψ_L and nucleon wave function φ. Within the quasi-three-body kinematics, these lead to

$$\Delta E_{nL} - i\Gamma_{nL}/2 = \frac{2\pi}{m_r} \sum_j \bar{a}_0^j \int d\boldsymbol{\rho}\, |\varphi(\boldsymbol{\rho})\psi_L(\beta\boldsymbol{\rho})|^2, \tag{7}$$

where summation over all nucleons is performed. The contribution of each nucleon is weighted by the overlap integral and energy averaged amplitudes

$$\bar{a}_0 = \int a_0(-E_s - E_a - P^2/(2\mu))|F_L(p)|^2\, dP. \tag{8}$$

The recoil energy is weighted by normalized Fourier transforms of $\varphi(\boldsymbol{\rho})\psi_L(\beta\boldsymbol{\rho})$, and $\beta = \frac{M_R}{M_R+M_N}$ reflects Jacobi coordinates used to describe the system.

The same procedure is applied to P wave interactions and related contribution to the atomic levels. We find that the dominant part is due to the P wave in the atomic wave function denoted as a P atom. Due to three body kinematics, there is also some small contribution due to the derivative of the nuclear wave function; mixed terms are negligible. Finally, we add a contribution to a non-mesic capture which contributes only to absorptive potential, denoted by NN. An experimental finding [31] is that it contributes 12% of all captures in Helium (see also [32]). The non-mesic interaction is expected to contribute a negligible shift, typical of large final phase space. Tables 4 and 5 compare all contributions.

Table 4. The table summarizes the results obtained by calculations of S wave interactions based on the Cieply and Smejkal model [23]. 2P level shifts and widths are given in eV. The last line provides the result from the Mazur scattering experiment.

Reference	Level Shift	Level Width
Wave	**Shift**	**Width**
S	−0.39	1.24
P atom	0.082	0.21
P nuclear	0.007	0.018
NN	0.00	0.146
Sum	−0.30	1.64
Mazur [17]	−0.09 (17)	0.94 (0.19)

Table 5. The table summarizes the results obtained by calculations of S wave interactions based on the Kyoto–Munich model [24]. 2P level shifts and widths are given in eV. The last line provides the result from the Mazur scattering experiment.

Reference	Level Shift	Level Width
Wave	**Shift**	**Width**
S	−0.336	0.798
P atom	0.082	0.21
P nuclear	0.007	0.018
NN [31]	0.00	0.136
Sum	−0.247	1.15
Mazur [17]	−0.09 (17)	0.94 (0.19)

3. Comparison to Data

Table 3 presents the S wave scattering amplitudes obtained according to several theoretical models. The reported numbers are extrapolated from the figures in ref. [33]. Next, the weighted averages $\bar{a}_0$ in region $E_{cm} = -34 \pm 6.6$ MeV are calculated. These amplitudes are used to calculate level shifts and widths of 2P levels, and the results are given in Table 6. The left part offers the first order of perturbation, the right part contains an approximate sum of multiple scattering series. It follows the method described in ref. [22] which, in the case of the three body system, is expected to yield precise results even with a divergent multiple scattering series.

Table 6. The results obtained by calculations with fixed $\Sigma(1385)$ parameters and the NN capture rate. S wave interactions scattering amplitudes are taken form several KN interaction models ((eV) units). The first two columns present the dominant first order. The last two columns include the approximate sum of higher multiple KN collisions.

Model	$shift_o$	$width_o$	$shift_{sum}$	$width_{sum}$
P [23]	−0.301	1.64	0.091	1.676
KM [24]	−0.247	1.15	−0.063	1.29
$M1$ [25]	−0.183	0.929	−0.079	1.05
$M2$ [25]	0.003	0.530	0.019	0.548
$B2$ [26]	0.200	0.516	0.211	0.482
$B4$ [26]	0.228	0.434	0.233	0.409
Mazur [17]	−0.09 (17)	0.94 (0.19)	−0.09 (17)	0.94 (0.19)

4. Discussion and Conclusions

The performed analysis favors amplitudes $M1$ of Guo and Oller [25] and KM of the Kyoto–Munich collaboration [24]. However, this does not confirm the general shape of the amplitudes in the whole region of $\Lambda(1405)$ but only the region of -34 MeV. Since Helium offers the most strongly bound valence nucleons of 21 MeV, it yields the lowest end of the energy region, which might be checked with X-ray transitions to "upper levels". The whole region, from $E_{cm} \sim 40$ up to $E_{cm} = 0$ MeV , has to be studied with heavier atoms.

Interesting are ($5g \rightarrow 4f$) in Al, Si, P, S, ($4f \rightarrow 3d$) in Be, B, C and ($3d \rightarrow 2P$) in Li. In the case of larger L values, the spread of recoil energy becomes smaller, and fixing the relevant E_{cm} region becomes sharper.

We find in Table 3 that K^- neutron interactions are not under control. Transitions to upper levels in ^{9}Be would be helpful in addition to K-Deuterium studies which are already underway [8]. These will study the amplitude in two non-overlapping energy regions.

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