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Does the Blackbody Radiation Spectrum Suggest an Intrinsic Structure of Photons?

Alex Khaneles

AT&T Inc., Middletown, NJ 07748, USA; akhaneles@att.com

Abstract: Photons are considered to be elementary bosons in the Standard Model. The assumption that photons are not elementary particles is assessed from an outlook of computational statistical mechanics. A prediction of variations in the shape of the blackbody radiation spectrum with polarization is made. A better understanding of the origins of quantum statistics could be crucial for theories beyond the Standard Model.

Keywords: foundations of quantum statistics; photon statistics; Leibniz's Principle of the Identity of Indiscernibles; theories beyond the Standard Model; cellular automata

1. Introduction

1.1. Motivation

In the last third of the nineteenth century Maxwell, Lord Kelvin, and Boltzmann came to the understanding of how irreversible macroscopic behavior arises from the time-reversible laws of microscopic physics. Though this was a great accomplishment, the resolution of apparent paradoxes remained controversial, and the foundations of statistical mechanics are still debated today [1]. At least some difficulties in the comprehension of how the logic of the micro level could be related to the macroscopic behavior were due to limitations of the mathematical/computational framework available at that time. This is where computer simulation of a system consisting of multiple identical elements could be helpful. If microdynamics is adequately defined, everything else would follow from it.

The second law of thermodynamics and the origins of the arrow of time have been illustrated with a reversible lattice gas cellular automata (CA) simulation. The substitution of individual atoms or molecules with one-bit objects on a discrete grid is simplistic, but the ability to provide a detailed description of a system—to keep track of every element without an information loss and actually reverse the evolution of the entire system—is quite unique. An expansion of such toy models can contribute to understanding more sophisticated statistical behavior [2]. CA are seen as a promising avenue to explore the roots of quantum mechanics [3,4] and build a deterministic underlying theory. As a step in this direction, an assumption that quantum statistics can be obtained without any fundamental randomness deserves to be examined.

The objective of this investigation was to construct a reversible working model *from the bottom up* that would exhibit the statistical behavior of massless bosons and generate the blackbody radiation (BBR) spectrum. It is widely believed that the time reversal invariance plays essentially the same role in classical and quantum theories. Some statistical features of Planck's quanta have been replicated by reversible CA [2], and it was expected that elements of Bose–Einstein (B–E) statistics could be implemented in the same method. However, the phase space of photons in B–E statistics—with a different number of cells for each species of quanta—cannot provide a uniform background required for a bijective function (would it be a unitary transformation or a reversible CA rule) and by that, it is not compatible with reversible time evolution. This was an obstacle that motivated the search for an alternative solution. One of the possible alternatives was to look at photons

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as composite particles instead and generate energy distributions for anticipated structures utilizing the same CA simulation base as in [2]. This approach is presented below.

The first theory of composite photons was proposed by de Broglie in 1932 [5]. It was a call for the reconciliation of photons with Maxwell's electrodynamics. Some of the latest fundamental unified theories beyond the Standard Model consider only fermions as fundamental particles [6,7] and all bosons to be composite.

Another reason to look at boson statistics from a different perspective came from ontology. According to Leibniz's Principle of the Identity of Indiscernibles (PII), there cannot be separate objects with all the same properties. In physics, classical particles are impenetrable and can be identified by trajectories. Fermions obey Pauli's Exclusion Principle and cannot have all their quantum numbers in common. They are "*weakly discernible*", and the PII still can apply [8,9]. On the contrary, the elementary bosons can have completely the same sets of properties and can be impossible to tell apart and defy the PII. Even so, individual photons are registered in experiments routinely the same way as other neutral composite particles—fermions or bosons. Are photons a good counterexample to the PII as elementary bosons? To what degree should Leibniz's principle be abandoned? Or, should B–E statistics be reevaluated? The lack of identity for quantum particles does not persuade philosophers. They are engaged in debates [10], and some are calling for a reinterpretation of the concepts [11,12].

1.2. Historical Annotations

In the derivation of his formula, Planck utilized the product of two factors: the spatial density of radiation energy (in parentheses) and the mean energy U_ν for "monochromatically vibrating resonators" of frequency ν [13,14]:

$$\mathrm{u}_\nu d\nu = \left(\frac{8\pi\nu^2}{\mathrm{c}^3}\right) U_\nu d\nu \tag{1}$$

The resonators can accommodate an integer number of "energy elements" $h\nu$, so the mean energy at temperature T is

$$U_\nu = \frac{h\nu}{e^{h\nu/k_B T} - 1} \tag{2}$$

The constants above are Boltzmann's $k_{B'}$, Planck's h, and the speed of light c. While the quantization ideas had flourished in a variety of physical applications, the indistinguishability of quanta as elements of energy brought questions [15], and the radiation density (number of resonators for each "spectral range") had not been understood in the same statistical terms as the mean energy.

It took more than 20 years before Bose invented new statistics with "different species of quanta each characterized by the number N_s and energy $h\nu_s$ (s = 0 to s = ∞)" [16,17]. He associated the quanta with frequency v_s with "a cylindrical surface" in a phase space and divided "the total phase space volume into cells of magnitude h^3" to obtain the number of cells for each frequency interval dv. Bose had arrived at the radiation density in (1) after multiplying the number of cells by a factor of 2 to take into account the polarization. Bose's derivation of Planck's formula was "obscure" and "only *a posteriori* justified" in Einstein's words [18], and Bose himself was not fully aware of his departure from classical statistics:

> I was not a statistician to the extent of really knowing that I was doing something which was really different from what Boltzmann would have done, from Boltzmann statistics. (as quoted in [18])

The desire to find the "real essence" of B–E statistics was one of the primary motivations behind Schrödinger's development of wave mechanics [19]. In 1926, Dirac incorporated B–E statistics into quantum mechanics and linked it with symmetric eigenfunctions [20]. Fowler offered a general form of statistical mechanics in which classical, B–E, and Fermi–Dirac statistics are special cases [21]. The term *photon* was coined the same year [22] and quickly became popular.

Pauli's Exclusion Principle was formulated for electrons in 1925. Later, Weyl recognized a connection between Pauli's principle and the PII. Pauli rejected Weyl's idea:

> This sounds like a *philosophical* principle and then, it seems to me, there are only two possibilities: a) as such it is wrong; b) it is correct, but *nothing* follows from it for physics . . . This would really be a strange principle in the philosophy of Leibniz, *which does not hold for all objects* (e.g., not for photons, as Weyl explicitly states) but only for *some* (as quoted in [23])

The history of quantum statistics and related philosophical questions are well presented in [24,25].

Quantum mechanics was extremely successful in calculating probabilities, but does it point to deterministic or random underlying processes? The Schrödinger equation describes the wave function evolution as deterministic and reversible. Nonetheless, the standard Copenhagen interpretation of quantum mechanics adopted a probabilistic explanation of the wave function (the Born rule, 1926) and its indeterministic and irreversible collapse on measurement. Lacking rationale for probabilistic rules was profoundly unsatisfying for some physicists. In Schrödinger's words, "everything ironed out and the true problems concealed" (as quoted in [26]). With the quick development of quantum theories, the physics community became more acceptive of new probabilistic/statistical ideas, and new suggestions could be postulated and used without strict causal vetting. When probabilities are primarily measured in experiments, it could be too difficult to recognize what is actually happening in the quantum world, and heuristic rules can be adopted. For example, in quantum electrodynamics:

> . . . the price of this great advancement of science is a retreat by physics to the position of being able to calculate only the *probability* that a photon will hit a detector, without offering a good model of how it actually happens . . . theoretical physics has given up on that [27]

The connection between spin (symmetrization of wavefunction) and statistics is seen as an empirical fact in non-relativistic quantum mechanics. In the pursuit of understanding the origins of quantum statistics, Fierz and Pauli came up with a justification in relativistic theory in 1939–40 [28]. However, as it was stated by Feynman,

> It appears to be one of the few places in physics where there is a rule which can be stated very simply, but for which no one has found a simple and easy explanation. The explanation is deep down in relativistic quantum mechanics. This probably means that we do not have a complete understanding of the fundamental principle involved. [29]

De Broglie pioneered the theory of composite photons in 1932. His photon consisted of two, then hypothetical, corpuscles: a neutrino and an anti-neutrino. Initially, several researchers went after the idea, but it did not find much traction afterward (see [5] and references therein). In quantum electrodynamics, which is now integrated into a more comprehensive theoretical set of the Standard Model, the photon is still elementary with no known persistent constituents. (Due to the uncertainty principle, any elementary particle in the quantum field theory, including a photon, can fluctuate into a variety of short-lived virtual states. If a virtual particle interacts with another object, it could expose the structure. The existence of such structure has been well established for photons experimentally at high energies [30].) However, de Broglie's idea may be reintroduced to reconsider the role that gauge bosons are playing within quantum field theory [31].

Different interpretations of quantum mechanics were developed over time (from de Broglie's pilot wave theory in 1927 to the modern versions of superdeterminism [3]). The quest for understanding quantum foundations is not over. In Einstein's belief, quantum mechanics is incomplete and should be revised:

> the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the

> framework of classical mechanics . . . it appears impracticable to give up this program in the "microscopic" alone. Nor can I see any occasion anywhere within the observable facts of the quantum-field for doing so, unless, indeed, one clings *a priori* to the thesis that the description of nature by the statistical scheme of quantum-mechanics is final. [32]

Computer simulation could be instrumental in connecting hypothetical microscopic structures with statistical distributions that can be measured experimentally.

2. Simulation of BBR

Massive particles of classical ideal gas exchange momentum and energy in reversible elastic collisions. These collisions alone can bring an isolated system to equilibrium. On the other hand, photons do not interact with each other under normal conditions. The mechanism of establishing equilibrium for BBR is emission and absorption of photons in cavity walls (equilibrium with matter). The walls could be regarded as a heat bath for radiation. The number of photons in a cavity at equilibrium can be considered to be constant. The preferred framework for the photon gas to explain BBR is B–E phase space, and thermalization redistributes photons between the phase space cells. Could this process be reversible like the collisions in an ideal gas and be in agreement with Boltzmann's ideas [1]?

2.1. The CA Model Where Particles Are Photons

The integer lattice gas automaton utilized in this investigation (see details in [2]) is based on ideas of continuation of motion (a particle moves in the same direction until it experiences a collision) and detailed balancing (in a head-on collision, particles are deflected perpendicularly). The output of such simulation is defined by initial conditions. Three parameters characterize the system as a whole and come from initialization: zero-point energy z, the mean occupation number $\overline{n}$ ($\overline{n} > z$), and the step in occupation numbers s. If the lattice sites are initialized with "elements of disorder", the evolution leads to statistical equilibrium, and the most probable exponential distribution is expected for the integer characteristics of the lattice sites.

In this model, structureless gas particles can be seen as photons of the same frequency (a single species of quanta). They are massless and have an assigned fixed energy. For a two-dimensional rectangular CA lattice, each lattice site contains four integers. Each integer accommodates a number of photons moving in one of four possible directions and can be identified with the resonator and/or phase space cell. The automaton redistributes the photons between the resonators/cells. There is no need for intermediate emission and absorption or cavity walls to thermalize photons. Such an isolated system is fully reversible and can be brought to equilibrium by itself. Planck's mean energy factor (2) is applicable to it.

If such a system could be expanded for multiple species of quanta, one could obtain a BBR spectrum. However, a necessary condition for reversibility is the conservation of information. It implies bijective uniform mapping from input lattice sites to the same number of output sites in the CA model. Such symmetrical one-to-one mapping cannot be performed between different numbers of phase space cells for two or more species of quanta. It would not be possible to establish bijective mapping through any intermediary either—like the cavity walls in the traditional understanding of photon thermalization. Thus, the B–E phase space logic for multiple species of quanta is not reversible and could not assist in building a reversible CA.

Indivisible energy is a property of the particle in the CA model and in order to comply with conservation of energy/momentum in each collision, the same value of energy should be assigned to all the particles. This is another restriction of the model that makes energy exchange impossible between different species of quanta.

With these two restrictions in mind, one can still extend the reversible integer lattice gas model. The lattice can be expanded from two into three dimensions (with six integer characteristics in each lattice site instead of four), or another set of four integers can be added to each site in a two-dimensional lattice. The added integers can be assigned to another "breed" of particles with the same energy to make an exchange with the first set possible. All the lattice sites are still uniformly connected with their neighbors, but the elements of the lattice gas model could obtain a different interpretation. The lattice site attributes can be treated as components of a composite structure. As an example, the next section describes how Einstein's theory of specific heat can be imitated with such a CA model.

2.2. Einstein's Specific Heat and Wien's Formula for BBR

Einstein made use of Planck's discontinuity of energy in his theory of specific heat. In his model of a solid, atoms oscillate independently with the same frequency in the three-dimensional lattice. Each of the three degrees of freedom in the oscillations can be associated with a resonator and corresponding mean energy (2). As a result, at low temperatures, the heat capacity is decreasing but at high temperatures, the mean energy per atom is still coming to the classical limit $\bar{\varepsilon} = 3k_BT$ (k_BT per degree of freedom).

The detailed energy distribution for atoms (triplets of resonators) in a solid is interesting in the context of this paper. It can be obtained from the integer lattice gas simulation (see Appendix A in [2]) or by using other methods, and it is shaped as Wien's distribution. (Wien came up with his empirical formula in 1896 to describe the BBR spectrum. With quick experimental progress, a call for reassessment came to Planck in 1900, and he improved Wien's formula.) To obtain the distribution from the CA simulation, all the integer characteristics of the three-dimensional lattice sites can be divided into three subsets: $E = \{e_1, e_2, \ldots, e_N\}$, $M = \{m_1, m_2, \ldots, m_N\}$, and $L = \{l_1, l_2, \ldots, l_N\}$, with N integers in each one. If the demonstration of reversibility is not a priority, a simple stochastic technique can be used instead of a CA simulation to generate the subsets (see Appendix A). While the integer lattice gas provides a working model of a system and the automaton is a reflection of microdynamics, it requires more computational resources and could have a long *relaxation time*. On the other hand, the Monte Carlo approach is a fast way of obtaining specific distributions while disregarding the cause. Either way, one can produce elements to assemble the composite structures. By combining integers from the subsets, a new variable, ε_i, can be introduced as follows:

$$\varepsilon_i = e_i + m_i + l_i \tag{3}$$

The energy of the triplets (3) produces Wien's distribution. The triplet can be seen as a composite structure with the elements of subsets E, M, L as constituents. In this interpretation, the lattice gas particles are not complete physical objects anymore (like the photons as they were interpreted in the previous section) but simple *energy bits* for the constituents. Each integer in (3) can be associated with a translational degree of freedom in Einstein's model of a solid. Every energy bit has the sole energy ε_z. Table 1 provides a short summary of the two interpretations of the lattice gas in Sections 2.1 and 2.2.

Table 1. Two interpretations of the integer lattice gas.

Caption	Section 2.1 (Basic)	Section 2.2 (Composite Structures)
particle	elementary photon	energy bit
integer characteristic of lattice site	resonator or phase space cell	resonator or a constituent of a composite object
number of particles in integer characteristic	number of photons associated with the resonator or phase space cell	energy of the resonator or constituent
good for	single mode of radiation; most probable distributions	detailed look at Einstein's specific heat; Wien's distribution
what is problematic	B–E phase space is not suitable for a reversible system; not consistent with the PII	

3. Photon Structure

The triplets (3) can generate Wien's distribution. Could a similar structure bring the energy distribution closer to the Planckian spectrum with the corresponding average energy per photon, $\bar{\varepsilon} \approx 2.7k_BT$? It is only about 10% less than for the triplet. A duplet of noninteracting constituents would not have sufficient average energy for BBR.

By examining statistics for different combinations of two numbers—one from each of the subsets, E and M—this study has found that the sum of both plus the geometric mean,

$$\varepsilon_i = e_i + m_i + \sqrt{e_i m_i}, \tag{4}$$

produces a distribution that is close in shape to Planck's law. It is presented in Figure 1. Planck's law function graph is for the fixed number of photons (Appendix B) to make a proper comparison to the energy spectrum of the same number of *constructed* photons (4). The temperature is the same in simulated exponential distributions for the subsets, E and M, and in function (A5). The units of energy ($\varepsilon_z = 1$) are the same for both.

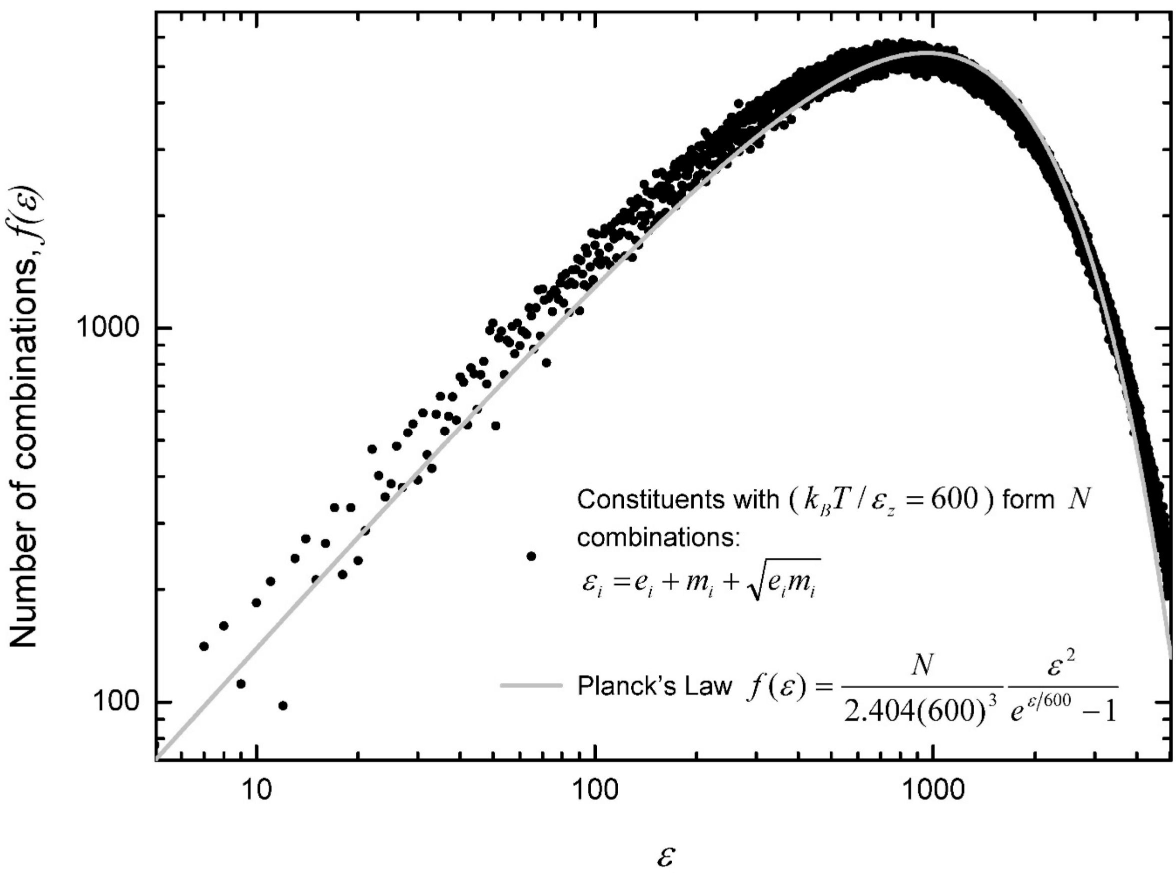

Figure 1. The number of combinations ($\varepsilon_i = e_i + m_i + \sqrt{e_i m_i}$) in the distribution is N. Planck's law function graph is for the same number of photons. Two arrays of constituents used in this simulation are composed of $N \sim 12 \times 10^6$ integers each. The simulation parameters are the mean occupation number $\overline{n} \approx k_BT/\varepsilon_z = 600$, zero-point energy $z = 1$, and the step in occupation numbers $s = 1$.

The energy quantization for the constituents would cause degradation of the energy distribution for composite photons at low mean occupation numbers that would become pronounced at $\overline{n} < 30$. Such degradation is not foreseen in the standard BBR explanation. To avoid this effect, the presented computations are for $\overline{n} \approx k_BT/\varepsilon_z = 600$. Other simulation parameters are $z = 1$ and $s = 1$. It is assumed that each constituent holds at least one energy bit (zero-point energy is equal to unity—no emptiness).

The two independent variables in (4) can be understood as two constituents of a photon for which energy can vary. The geometric mean can be seen as interaction energy between the two or as rotational energy. It is fully defined by the energy of two constituents and does not come as another degree of freedom. From the perspective of the structure, the average energy per photon in the BBR spectrum, $\bar{\varepsilon} \approx 2.7k_BT$, comprises the average energy of two constituents, $2k_BT$, and the interaction/rotation, ~$0.7k_BT$. If energy is defined as the total number of particles in the CA system, like was done before this section, the conservation of energy is not in question. On the other hand, the total interaction/rotational energy for the system in this section, $\sum_N \sqrt{e_i m_i}$, will fluctuate with evolution steps.

The CA rule updates all cells in one step, and the steps form a succession. It can be seen as working in the discrete time domain where the logic of events is not spatial. The CA grid is organized into a number of dimensions but does not provide continuous symmetries or other features one would expect from a fully-fledged continuous space or spacetime. The Monte Carlo simulation (Appendix A) ignores any spatial ideas altogether. However, the physical phenomenon of BBR and Planck's law defines not only the shape of energy distribution but the number of photons in the unit volume for the temperature (A4). It can be used to introduce distance and volume as secondary attributes into the system.

The introduction of spatial elements into the CA model was not the goal of this investigation, and such a possibility is discussed only in this paragraph. The system can be "*spatially extended*" [33] by assigning length to connections to neighboring cells in the CA grid. According to (A4), one photon on average takes the volume of a cubic box with a side length proportional to $\frac{hc}{k_BT}$. The length (distance between cells) can be attributed to a photon in this model and defined as the function—$d = \frac{w}{\varepsilon_i}$, where w is a constant—to satisfy spatial requirements of BBR. This function is analogous to the photon's wavelength–energy relationship in conventional terms: $\lambda = \frac{hc}{\varepsilon}$. The spatial locations of cells in such a system are relative and would fluctuate with changes in the energy of each cell with evolution steps. The assumption of nonzero positive minimal energy for photons is required for this distance function. Physical space in this model is curved by radiation like it is curved by masses in General Relativity, and there is no "empty" space. A similar assumption of positive minimal energy was made for the constituents to form energy distribution in Figure 1. The CA models are considered as candidates for emergent space of fundamental physics [3,4].

The two constituents, with a quite arbitrarily injected interaction/rotation, bring the energy distribution close to Planck's radiation law over a broad spectral range. The interaction could point to a force between the constituents but it does not originate from the generic microdynamics used to produce the constituents. This structure is reminiscent of mesons in the Standard Model: one quark and one antiquark bound together by the strong interaction. De Broglie's attempt to reconcile photons with Maxwell's electrodynamics brought him to the first composite photon theory. This paper represents another search for continuity, now in statistics, but again, points to a structure that is akin to de Broglie's.

4. Polarization and a Hypothetical Experiment

BBR, in general, is unpolarized. In the conventional explanation of the BBR spectrum, spin or polarization is taken into account as a factor that doubles the number of photons but does not affect the shape of energy distribution. On the other hand, the composite structure (4) has two components (like the electromagnetic field in classical electrodynamics) and, in addition, it incorporates interaction/rotational energy that affects the shape of distribution without bringing in another degree of freedom, like spin or polarization.

However, it makes sense to consider an additional degree of freedom for the rotation of a composite photon. It has been shown, for example, in optomechanical measurements of photon spin angular momentum and optical torque [34], that the sign and magnitude of optical torque are determined by the photon polarization states. The effective spin angular momentum of a photon is equal to zero for linearly polarized light and is significant when the polarization is circular or elliptical. The same impact from polarization can be expected on the rotational energy of photons. In (4), $\sqrt{e_i m_i}$ would represent the mean rotational energy for a subset of unpolarized photons with specific energy of the constituents, e_i and

m_i. The rotational energy could vary with polarization and, for linearly polarized light, would be always zero. If polarization is constrained, the shape of energy distribution would differ from Planckian as well. For linearly polarized light (a duplet $\varepsilon_i = e_i + m_i$ with no rotation), the energy distribution would be shaped as $A\frac{\varepsilon^2}{e^{\varepsilon/k_B T}}$, where A is a constant. This prediction for the shape of the energy spectrum could be tested experimentally.

One of the hypothetical experiments to detect the composite structure of photons could be similar to a measurement of the BBR spectrum in a well-controlled laboratory environment. The typical system consists of a BBR source, a spectrometer, and a detector (a photo sensor to measure the light intensity). It can be supplemented with a polarizer installed between the BBR source and the spectrometer. With the polarizer, the shape of the measured distribution would differ from Planckian as predicted above—this would support the suggestion of this paper. An optical setup in [35] can be seen as an example of a system for precisely measuring thermal radiation spectra. Additional experimental details on how to integrate a polarizer into such a system can be found in [36].

5. Conclusions

B–E statistics was first introduced specifically to explain Planck's law. Further advances in particle physics discovered multiple other bosons in addition to photons. The majority of all bosons are believed to be constructed from an even number of quarks/antiquarks or other particles, while photons, along with other gauge bosons and Higgs bosons, are still regarded as elementary particles in the Standard Model. A possible alternative justification of the Planckian spectrum can come from the intrinsic structure. If one accepts the BBR spectrum as a manifestation of the photon's structure, one might also infer that other fundamental bosons have a similar intrinsic organization and are not elementary.

If all bosons were not elementary, Leibniz's Principle of the Identity of Indiscernibles would be vindicated in particle physics since "the only cases in which the status of quantum particles as objects is seriously in question are ... *elementary* bosons – bosons (supposedly) with no internal fermionic structure" [8].

The understanding of quantum statistics' origins plays a key role in building fundamental unified theories beyond the Standard Model. In supersymmetry, bosons and fermions are treated as fundamental particles. Each fundamental particle from one group has an associated so-called "superpartner" in another group. Supersymmetry predicts a large number of undiscovered elementary particles. With no fundamental bosons, there would not be a rationale for this kind of symmetry. Some alternative theories (like Spinor Gravity [6] and Causal Fermion Systems [7]) see the fermions as fundamental particles and the bosons as composite objects. Accelerator experiments in high energy physics are costly and take years. Any other experimental methods to test these theories should be considered.

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Appendix A. Stochastic Simulation

A Monte Carlo technique similar to the one used in numerical integration can be deployed to populate arrays (or sets) of N integers and obtain the distribution of a given shape $y = f(x)$ for those numbers. To do so, pairs of real pseudorandom numbers (Rx and Ry) can be generated as points in the rectangular region that entirely covers the function graph $y = f(x)$. If Ry falls below $f(Rx)$, the value Rx is rounded up to the nearest integer and populates one element in the array of integers. Otherwise, the pair of pseudorandom numbers is discarded. The cycle is repeated until all elements of the array are filled.

Each array would form a most probable (Boltzmann) distribution defined by the exponential function $y = f(x) = e^{-x\varepsilon_z/k_B T}$. The integer would stand for the number of energy bits in the constituent of the composite structure. The rounding up makes the spectrum discrete and sets its minimum to unity.

Appendix B. Planck's Radiation Law for a Fixed Number of Photons

With photon energy ε instead of $h\nu$, Planck's law for a unit volume can be written as follows:

$$u(T,\varepsilon) = \frac{8\pi}{(hc)^3}\frac{\varepsilon^3}{e^{\varepsilon/k_B T}-1} \tag{A1}$$

The corresponding number of photons is distributed with energy as follows:

$$n(T,\varepsilon) = \frac{u(T,\varepsilon)}{\varepsilon} = \frac{8\pi}{(hc)^3}\frac{\varepsilon^2}{e^{\varepsilon/k_B T}-1} \tag{A2}$$

The total number of photons in the unit volume at temperature T can be found from integration

$$N_0 = \int_0^\infty n(T,\varepsilon)d\varepsilon = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{\varepsilon^2 d\varepsilon}{e^{\varepsilon/k_B T}-1}$$

Let $x = \varepsilon/k_B T$ and take into account that

$$\int_0^\infty \frac{x^2 dx}{e^x - 1} = 2\zeta(3) \approx 2.404 \tag{A3}$$

where $\zeta(3)$ is the Riemann zeta function, also known as Apéry's constant. (The stochastic procedure, like the one in Appendix A, could be utilized to compute this integral as well). This results in the spatial factor in Planck's law

$$N_0 \approx 2.404\frac{8\pi(k_B T)^3}{(hc)^3} \tag{A4}$$

with the fixed number of photons, N, distributed by energy as follows:

$$f(T,\varepsilon) = \frac{N}{N_0}n(T,\varepsilon) \approx \frac{N}{2.404(k_B T)^3}\frac{\varepsilon^2}{e^{\varepsilon/k_B T}-1} \tag{A5}$$

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