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Designing a Laboratory Cam Profile Measuring Machine to Validate Follower Displacements

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Abstract: This study presents the design, build, and evaluation of a laboratory cam profile measuring machine tailored to demonstrate the mechanical principles and applications of various cam shapes. Utilizing a diverse set of cam profiles, the machine effectively converts rotational motion into measurable linear motion, achieving a range of motion profiles, including rising, declining, steady, and instantaneous actions. Key components of the machine include an angle gauge for precise rotational measurements and a linear dial indicator for accurately gauging the cam-induced displacement. This setup facilitates the measuring of displacement, and computation of velocity and acceleration for each cam shape, offering a dynamic visual and numerical aid for engineering and design.

Keywords: cam design; cam lift measurement; follower displacement; additive manufacturing; machine design; laboratory equipment; data collection

1. Introduction

A cam is a non-uniform circular profile that, when employed with a follower—such as a pin or a roller tracing the cam's outer edge—converts rotational motion into linear motion. Cams find applications in designs requiring precise timing. Rotated at a constant speed, cams enable specific events to occur at predefined intervals. The design and structure of a cam determine whether the resulting linear motion is ascending, descending, stable, or instantaneous [1].

If the follower loses contact with the cam, chaos is created in the follower's motion. As presented in a study by Yousuf, a flat-faced follower was used to contact a cam [2]. The follower velocity was plotted against the displacement at different rotational velocities. As the rotational speed increased, the movement of the follower became less periodic and more chaotic. The shape of the cam and the design of the cam profile affect the motion of the follower. The cycloidal motion used in one of the cams in this research promotes follower contact with the cam by eliminating the jerk experienced by the follower.

In the analysis of cam mechanisms, follower contact and movement is as important as the cam design itself. The follower's degrees of freedom must be restricted to limit undesired movements. In an experimental setup at San Diego State University, multiple guides were used to restrict the follower's movement to one axis. The setup used two guides to restrict the movement to the vertical axis. A flat-faced follower was used to contact the rotating cam. Data were collected using a series of high-speed cameras to track the follower movement. The experimental setup was able to successfully track the movement of five spring mass damper systems connected in series attached to the follower [3].

A dwell–rise–dwell cam, referenced as the double dwell cam, was designed and measured in this study. Dwells can be found in many different cam mechanisms and are a fundamental part of cam design. Dwell–rise–dwell or rise–dwell–return cams have been used in previous research to explore beta functions and their ability to approximate the motion of a pivoting cam follower [4].

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Creating cams from desired follower displacement is not a new technique. In research by Sun and Tang, cam models were developed in Pro-E using an algorithm they created. The program outputted the cam profile based on the desired follower displacement. Simulations in Pro-E were conducted to verify the follower displacement created by the cam. The results aligned with the original curves inputted into the algorithm [5].

The motion of the follower and its lift dependence create complex equations when time and varying rotational velocity are involved. To simplify the equations and avoid solving the differential equations yielding displacement, velocity, and acceleration, constant angular velocity was assumed in an analysis by Hejma [6]. The same methodology was utilized in this study to simplify the equations describing the motion of the followers.

Although cams and cam design have been around for decades, new applications of cams are still being created. A new approach to continuously variable transmissions (CVT) utilizes two cams alternating pushing on actuating rods. The actuating rods apply force to an oscillatory slotted link with an adjustable pivot. Moving the pivot of the slotted links creates infinitely adjustable gear ratios. The slotted links push and pull on ratcheting grooved wheels to transfer the motion back to rotational. The benefits of the belt-less CVTs are the amount of torque that can be transmitted through the system. Traditional CVTs will have belt-slip when high torque situations are encountered. The system presented by Al-Hamood provides a theoretical solution to a primary problem with current CVTs [7].

The profile of cams can be complex and elaborate depending on the desired motion of the follower. Often the motion can not be accurately predicted from a visual inspection alone. Motion Simulation software exists but may be out of budget for low-use applications and lacks precision when measuring intricate or complex shapes. An accurate laboratory cam profile measuring machine needs to be utilized to verify the expected follower motion with the actual follower motion. Using prior research, Computer-Aided Design (CAD) software, and a 3D printer, a cam measuring machine was designed and fabricated to measure cam profiles in this research study.

2. Design

All cams investigated here have the same general requirements of a minimum radius of 1 inch, a maximum radius of 2 inches, and a maximum lift of 1 inch. Designing a cam shape necessitates determining displacement at different angles. Initially, a graph of displacement versus angle in polar coordinates is generated to reflect the desired cam follower motion. This graph is then translated into parametric equations within the Cartesian coordinate system, effectively transforming the displacement curve into a physical shape. Assuming continuity in the displacement curve, the resulting shape represents the cam's profile, which yields the original displacement curve. Assuming angular velocity is constant, the derivative of displacement versus angle will yield velocity versus angle. Similarly, the derivative of velocity versus angle will yield acceleration versus angle.

In this research on cam design and profile, the angle is referred to as θ and has units of degrees. The maximum follower lift uses the variable h and has units of inches. The maximum and minimum radii of the cam are referred to as $R_{\max}$ and $R_{\min}$, respectively. Both radii variables have units in inches. Formulas for displacement, velocity, and acceleration follow the common convention of d, v, and a, respectively. These variables were used in the equations below to describe the motion, profile, and expected results of the cams.

The purpose of a laboratory cam profile-measuring machine is to measure the effects of cam shapes on the cam follower displacements. To demonstrate the effect, common types of cams are selected for design and manufacturing. The selected profiles can be modeled and 3D printed. The 3D-printed cams allow for experiments to be conducted to demonstrate each cam profile's operation and unique purpose. Alternative machines such as the Tecquipment TM-1021 [8], IndiaMart TOM-02 [9], and Sun LabTek DOM-002 [10] can be used to capture data but are built for commercial use, which minimizes the availability for small-scale research or design. These machines are priced around 4000 USD. Based on price, the laboratory cam profile-measuring machine, as shown in Figures 1 and 2, can be fabricated for approximately 250 USD.

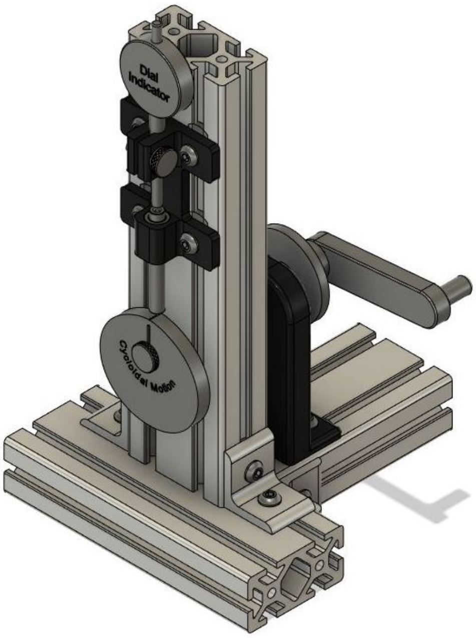

Figure 1. Computer-aided design (CAD) model of cam profile measuring machine, front view.

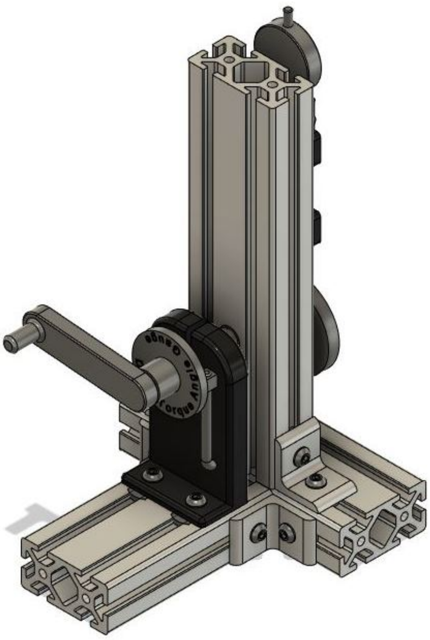

Figure 2. Computer-aided design (CAD) model of cam profile measuring machine, rear view.

2.1. Double Dwell

For the "Double Dwell" cam profile, the follower's displacement remains constant for 90 degrees at both the top and bottom of the follower's stroke, and the motion connecting the two dwells must have a linear follower displacement with respect to the rotational angle of the cam displayed in Figure 3.

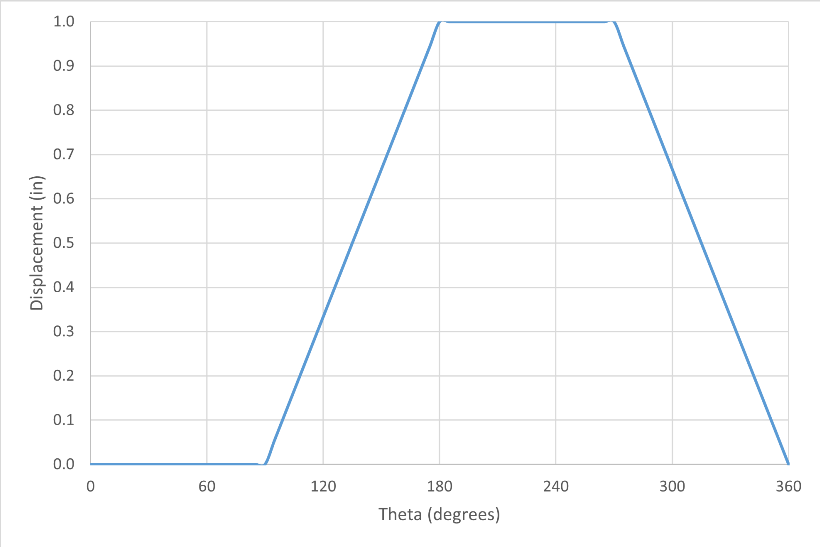

Figure 3. Theoretical double-dwell displacement.

A piece-wise function Equation (1) containing four regions was generated from the displacement vs. angle graph. Each region of the piece-wise function describes the motion within the region. The derivative of the displacement in Equation (1) can be taken, yielding the velocity in Equation (2). The derivative of the velocity in Equation (2) can be taken to yield the acceleration in Equation (3). These sets of equations were used to create the cam and verify the shape in the experimental results. The equations listed in Equation (1) are in terms of polar coordinates. Parametric Equations (4) and (5) were derived from Equation (1) and used in SolidWorks to generate the cam's profile. Figure 4 represents the theoretical shape of the cam generated from the displacement graph in Figure 3 and polar displacement Equation (1). The orange circle in Figure 4 represents the center of rotation. This convention is utilized for all subsequent cam shape graphs.

$$d(\theta) = \begin{cases} 0 & \text{for } 0 \leq \theta \leq 90 \\ (\theta - 90)\left(\frac{h}{90}\right) & \text{for } 90 < \theta \leq 180 \\ 90\left(\frac{h}{90}\right) & \text{for } 180 < \theta \leq 270 \\ (360 - \theta)\left(\frac{h}{90}\right) & \text{for } 270 < \theta \leq 360 \end{cases} \tag{1}$$

$$v(\theta) = \begin{cases} 0 & \text{for } 0 \leq \theta \leq 90 \\ \frac{h}{90} & \text{for } 90 < \theta \leq 180 \\ 0 & \text{for } 180 < \theta \leq 270 \\ \frac{-h}{90} & \text{for } 270 < \theta \leq 360 \end{cases} \tag{2}$$

$$a(\theta) = 0 \quad \text{for } 0 \leq \theta \leq 360 \tag{3}$$

$$x(\theta) = \begin{cases} (0 + R_{\min})cos(\frac{\pi\theta}{180}) & \text{for } 0 \leq \theta \leq 90 \\ (\theta - 90)(\frac{h}{90}) + cos(\frac{\pi\theta}{180})R_{\min} & \text{for } 90 < \theta \leq 180 \\ (90(\frac{h}{90})) + cos(\frac{\pi\theta}{180})R_{\min} & \text{for } 180 < \theta \leq 270 \\ (360 - \theta)(\frac{h}{90}) + cos(\frac{\pi\theta}{180})R_{\min} & \text{for } 270 < \theta \leq 360 \end{cases} \tag{4}$$

$$y(\theta) = \begin{cases} (0 + R_{\min})sin(\frac{\pi\theta}{180}) & \text{for } 0 \leq \theta \leq 90 \\ (\theta - 90)(\frac{h}{90}) + sin(\frac{\pi\theta}{180})R_{\min} & \text{for } 90 < \theta \leq 180 \\ (90(\frac{h}{90})) + sin(\frac{\pi\theta}{180})R_{\min} & \text{for } 180 < \theta \leq 270 \\ (360 - \theta)(\frac{h}{90}) + sin(\frac{\pi\theta}{180})R_{\min} & \text{for } 270 < \theta \leq 360 \end{cases} \tag{5}$$

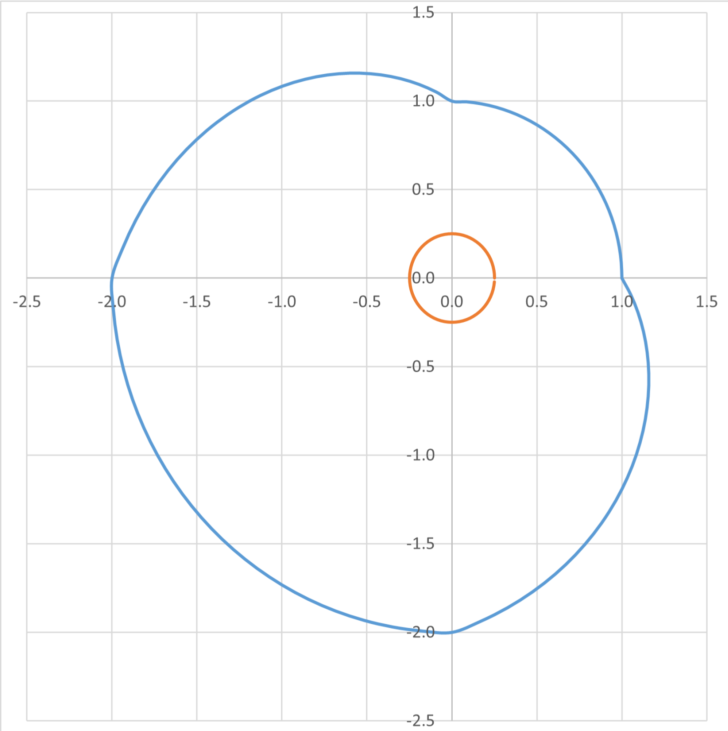

Figure 4. Double-dwell cam shape.

2.2. Linear Motion (Up Only)

The "linear motion (up only)" cam has a follower displacement that constantly increases for the entire 360-degree rotation of the cam before abruptly returning to the start. Figure 5 shows the directly proportional relationship between displacement and angle. The relationship creates a straight line that can be described using Equation (6). The derivative of the displacement in Equation (6) can be taken, yielding the velocity in Equation (7). The derivative of the velocity in Equation (7) can be taken to yield the acceleration in Equation (8). Since the displacement is directly proportional to the angle of the cam, the velocity is a constant, and the acceleration is 0. The cam is often called a "snail cam" because of the resemblance to a snail's shell that can be shown in Figure 6.

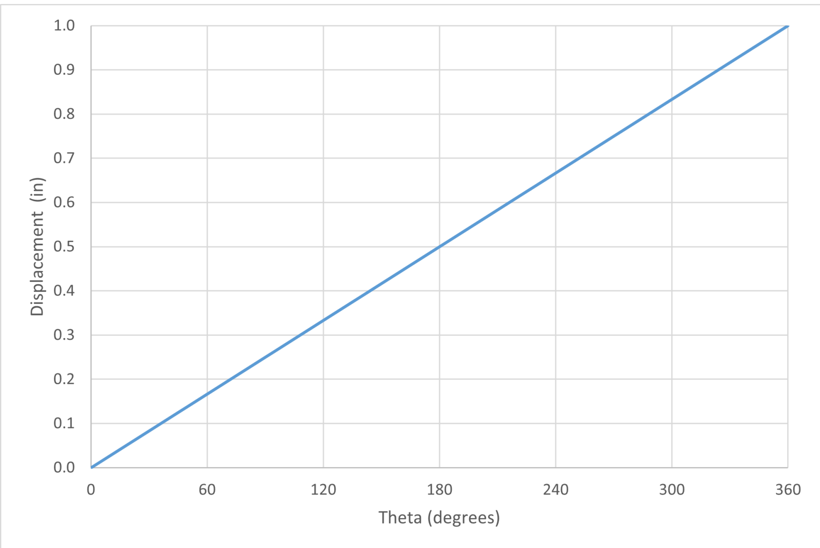

Figure 5. Theoretical linear motion (up only) displacement.

$$d(\theta) = \frac{h\theta}{360} \quad \text{for } 0 \leq \theta \leq 360 \tag{6}$$

$$v(\theta) = \frac{h}{360} \quad \text{for } 0 \leq \theta \leq 360 \tag{7}$$

$$a(\theta) = 0 \quad \text{for } 0 \leq \theta \leq 360 \tag{8}$$

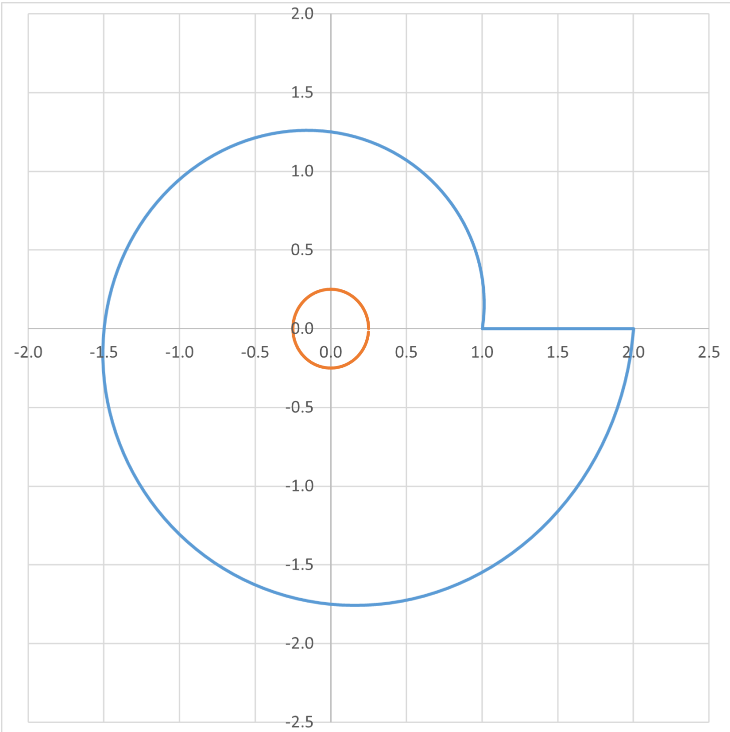

Figure 6. Linear motion (up only) cam shape.

2.3. Linear Motion (Up and Down)

The "Linear Motion (Up and Down)" builds on the principles of the Linear Motion (Up Only) in Section 2.2 using a directly proportional relationship between follower displacement and cam angle. However, this cam only has a positive relationship for the first 180 degrees, unlike the Linear (Up Only), which has a positive relationship for the entire 360 degrees. The remaining 180 to 360 degrees trend down, or has a negative linear relationship placing the end point at the starting point. Figure 7 demonstrates the increasing and then decreasing displacement with respect to the angle of the cam. A piece-wise function with two unique regions was used to describe the motion for displacement shown in Equation (9). The derivative of the displacement in Equation (9) can be taken, yielding the velocity in Equation (10). The derivative of the velocity in Equation (10) can be taken to yield the acceleration in Equation (11). The equations follow a similar trend of the "linear up" cam profile, being that the velocity is constant; however, the value changes between positive and negative depending on the region. The acceleration is 0 for the entire region of the cam, similar to the linear (Up Only) cam. Figure 8 visualizes the theoretical shape of the cam based on the polar displacement equations described in Equation (6).

$$d(\theta) = \begin{cases} \frac{h\theta}{180} & \text{for } 0 \leq \theta \leq 180 \\ \left(\frac{-h\theta}{180}\right) + R_{\max} & \text{for } 180 < \theta \leq 360 \end{cases} \tag{9}$$

$$v(\theta) = \begin{cases} \frac{h}{180} & \text{for } 0 \le \theta \le 180 \\ \left(\frac{-h}{180}\right) & \text{for } 180 < \theta \le 360 \end{cases} \tag{10}$$

$$a(\theta) = 0 \quad \text{for } 0 \le \theta \le 360 \tag{11}$$

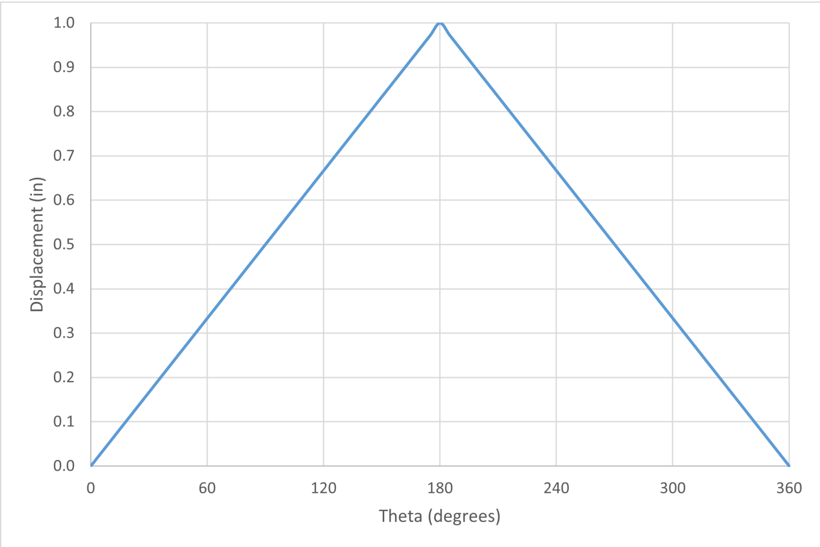

Figure 7. Theoretical linear motion (up and down) displacement.

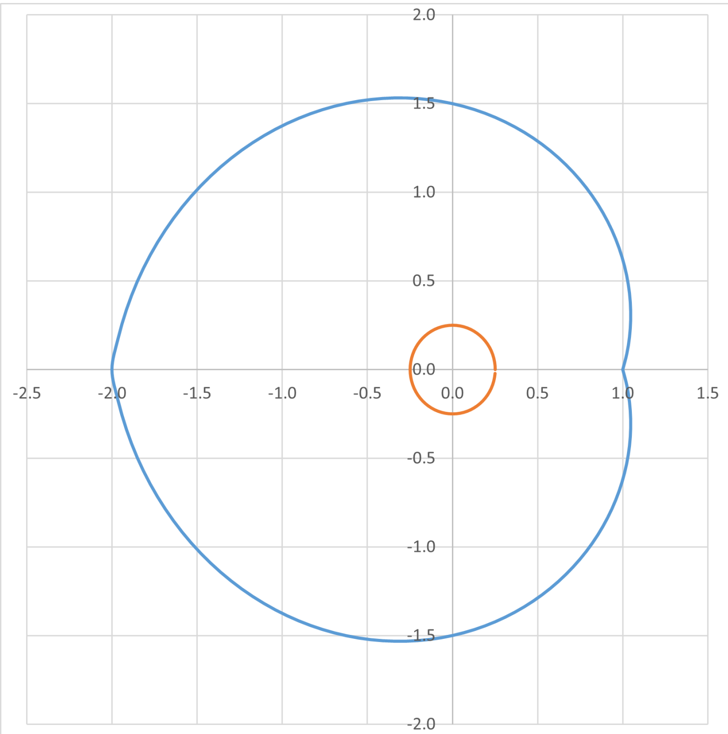

Figure 8. Linear motion (up and down) cam shape.

2.4. Harmonic Motion

The "Harmonic Motion" cam is based on the simple harmonic motion of a spring mass system. The spring mass system can be described using an ordinary differential equation (ODE). The general solution to the ODE for displacement has periodic motion and can be shown in Equation (12). In this equation, a conversion from radians to degrees and an accommodation for the desired lift and maximum radius were utilized. The derivative of the displacement in Equation (12) can be taken, yielding the velocity in Equation (13). The derivative of the velocity in Equation (13) can be taken to yield the acceleration in Equation (14). The acceleration equation is expected to be non-zero, unlike the previous cam profiles. Figure 9 is the follower displacement curve that was used to generate the cam shape shown in Figure 10.

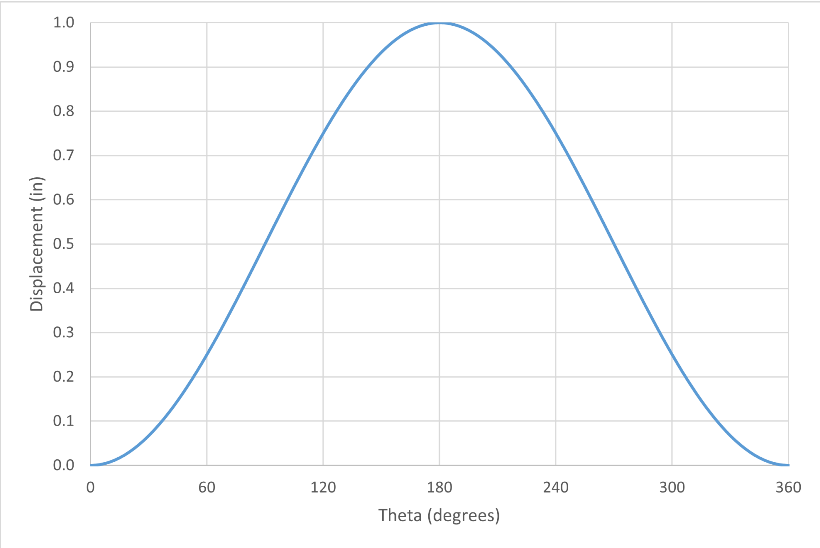

Figure 9. Theoretical harmonic motion displacement.

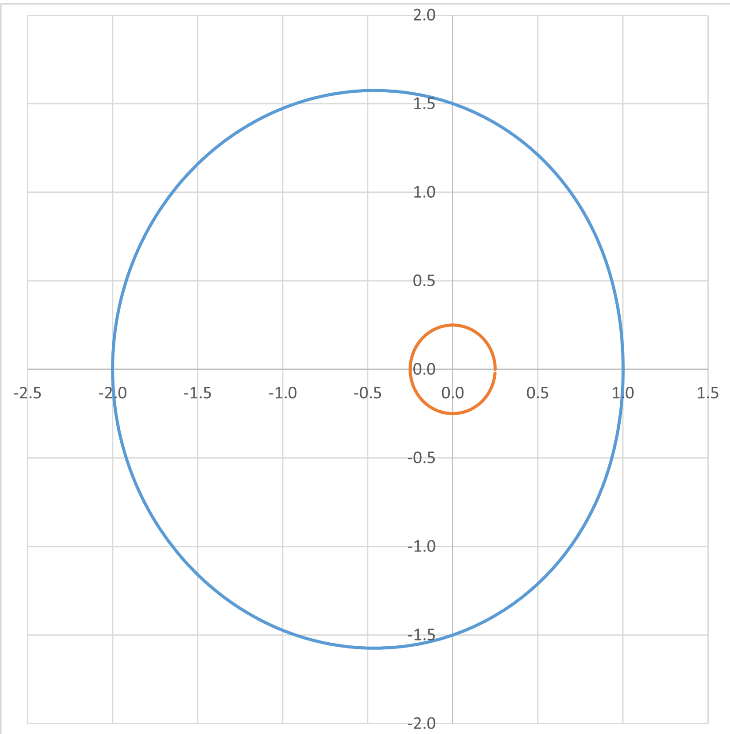

Figure 10. Harmonic motion cam shape.

$$d(\theta) = \frac{h}{2}(1 - cos(\frac{\pi\theta}{180})) \quad \text{for } 0 \leq \theta \leq 360 \tag{12}$$

$$v(\theta) = \frac{1}{360}\pi h \, sin(\frac{\pi\theta}{180}) \quad \text{for } 0 \leq \theta \leq 360 \tag{13}$$

$$a(\theta) = \frac{1}{64800}\pi^2 h \, cos(\frac{\pi\theta}{180}) \quad \text{for } 0 \leq \theta \leq 360 \tag{14}$$

2.5. Cycloidal Motion

The "Cycloidal Motion" cam demonstrates one of the most important profiles in cam design. The cycloidal curve is extremely useful for creating acceleration without jerk. Figure 11 displays the follower displacement in relation to the angle of the cam. The graph is similar to Figure 9; however, the point of the maximum displacement is more of a plateau than a peak. A piece-wise function was used to describe the motion for displacement in Equation (15). The displacement equations include a conversion from radians to degrees and amplitude modification to restrict the lift and maximum radius. The derivative of the displacement in Equation (15) can be taken, yielding the velocity in Equation (16). The derivative of velocity in Equation (16) can be taken to yield the acceleration in Equation (17). Figure 12 represents the theoretical cam shape that produces the displacement graph shown in Figure 11.

$$d(\theta) = \begin{cases} \frac{h}{\pi}(\frac{\pi\theta}{180} - 0.5sin(\frac{\pi\theta}{90}) & \text{for } 0 \leq \theta \leq 180 \\ \frac{-h}{\pi}(\frac{\pi\theta}{180} - 0.5sin(\frac{\pi\theta}{90}) + R_{\max} & \text{for } 180 < \theta \leq 360 \end{cases} \tag{15}$$

$$v(\theta) = \begin{cases} \frac{h}{180}(1 - cos(\frac{\pi\theta}{90}) & \text{for } 0 \leq \theta \leq 180 \\ \frac{h}{180}(cos(\frac{\pi\theta}{90} - 1) & \text{for } 180 < \theta \leq 360 \end{cases} \tag{16}$$

$$a(\theta) = \begin{cases} \frac{h}{16200}\pi sin(\frac{\pi\theta}{90}) & \text{for } 0 \leq \theta \leq 180 \\ \frac{-h}{16200}\pi sin(\frac{\pi\theta}{90}) & \text{for } 180 < \theta \leq 360 \end{cases} \tag{17}$$

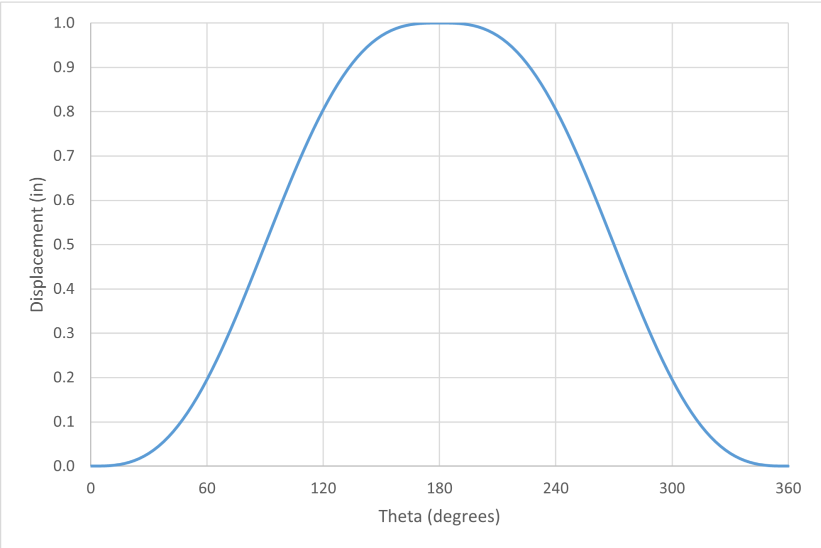

Figure 11. Theoretical cycloidal motion displacement.

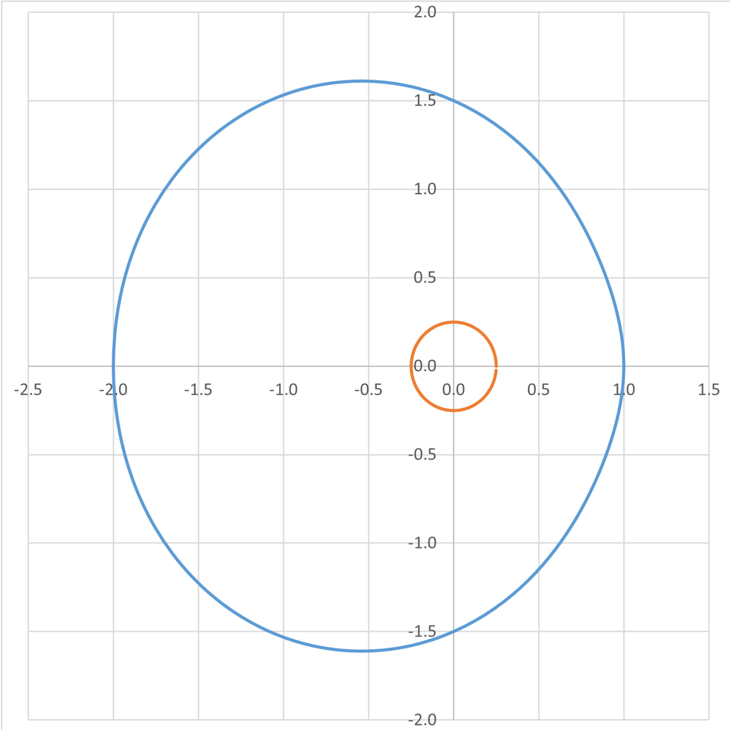

Figure 12. Cycloidal motion cam shape.

3. Build Instructions

The cam shapes chosen for the experiments were double dwell, linear motion (Up Only), linear motion (Up and Down), harmonic motion, and cycloidal motion, as shown in Section 2. The minimum and maximum radius bounds were established to complement the maximum measuring range of a 0″–1″ dial indicator. Each cam had a 3/8″ hole through the center so they could be mounted to a measuring device. The through hole is represented in all of the cam shape figures by an orange circle.

Once all the cam shapes were designed and their equations were verified, the parametric equations were entered into SolidWorks. Using the equation-driven curve function, the parametric equations were drawn in a 2D sketch plane. The lines were connected to maintain continuity and then extruded into a 0.395″ thick shape. From there, the model was sent to the 3D printer to manufacture the cam. All of the 3D-printed cams are shown in Figure 13. After all the cam shapes had been 3D printed, they were experimentally tested and compared to the theoretical displacement curves.

The laboratory cam profile measuring machine was designed in Fusion 360 and manufactured using purchased, machined, and 3D-printed parts. The complete machine is displayed in Figure 14. A complete drawing packet with a Bill of Materials is shown in Appendix A. The frame of the machine was constructed from 80/20 aluminum extrusion. The measuring devices (angle gauge and dial indicator) were purchased. The crank handle and holders for the angle gauge and dial indicator were 3D printed. The dial indicator holder utilizes a 3/8″ ID linear bearing to ensure free movement of the follower and that the central axis of the follower is colinear with the dial indicator. The followers were machined using 6061-T6 aluminum. The point follower used for this experiment maintained a plus or minus 0.0005″ outer diameter. The face contacting the cam had a large 45-degree chamfer on the outer edge, resulting in a point. A small filet was placed on the tip of the follower to promote the longevity of both the 3D-printed cams and the follower. The opposite face of the follower contacting the dial indicator was center drilled to provide a location for the tip of the dial indicator and a second method of support for the follower. The holders are secured to the frame using the T-Slots on the 80/20 allowing for easy adjustments and configuration changes, as shown in Figure 15.

The cams were mounted to a rotating shaft supported by bearings. The cams were secured to the shaft with a thumb screw that presses the cam between a fixed flange on the shaft and the face of the thumb screw creating friction and preventing slipping. The shaft was connected to the angle gauge at the rear of the machine displayed in Figure 16. The angle gauge was driven by a 3D-printed handle as shown in Figure 17.

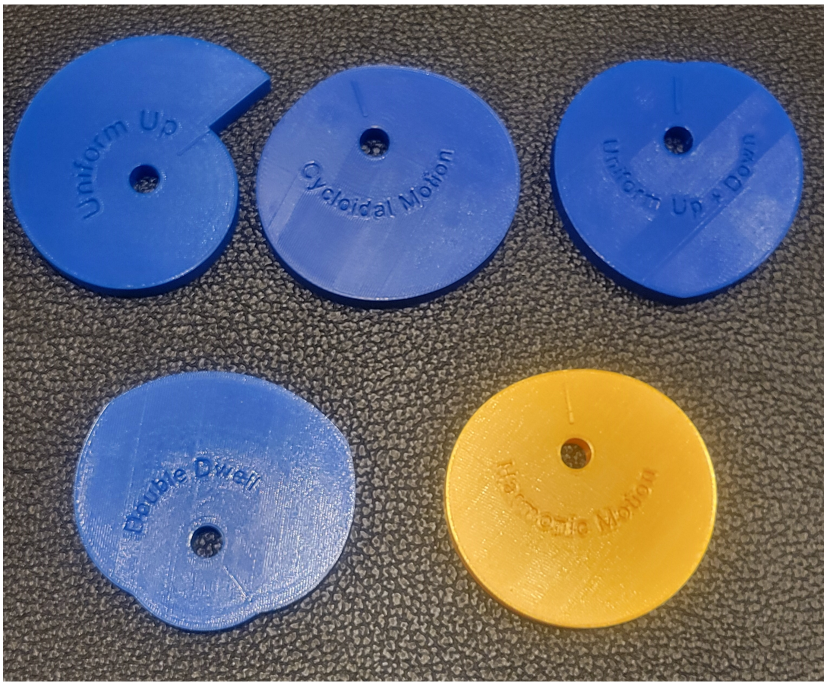

Figure 13. The 3D-printed cams.

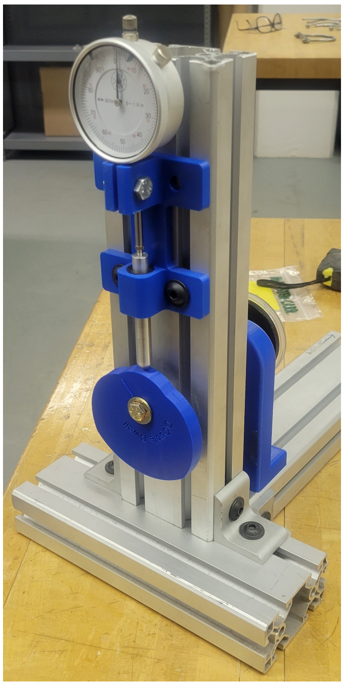

Figure 14. Cam shape and dial indicator.

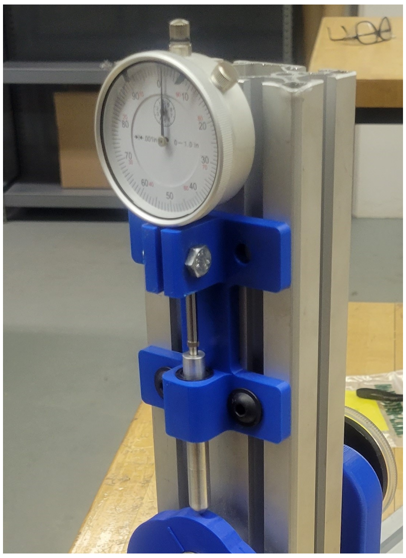

Figure 15. Point follower and dial indicator.

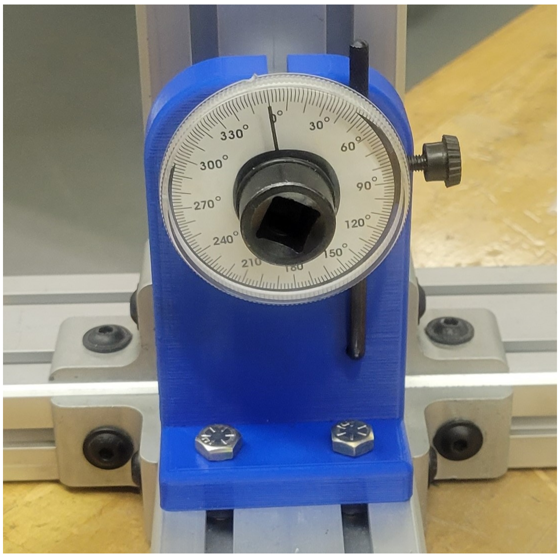

Figure 16. Angle gauge without handle.

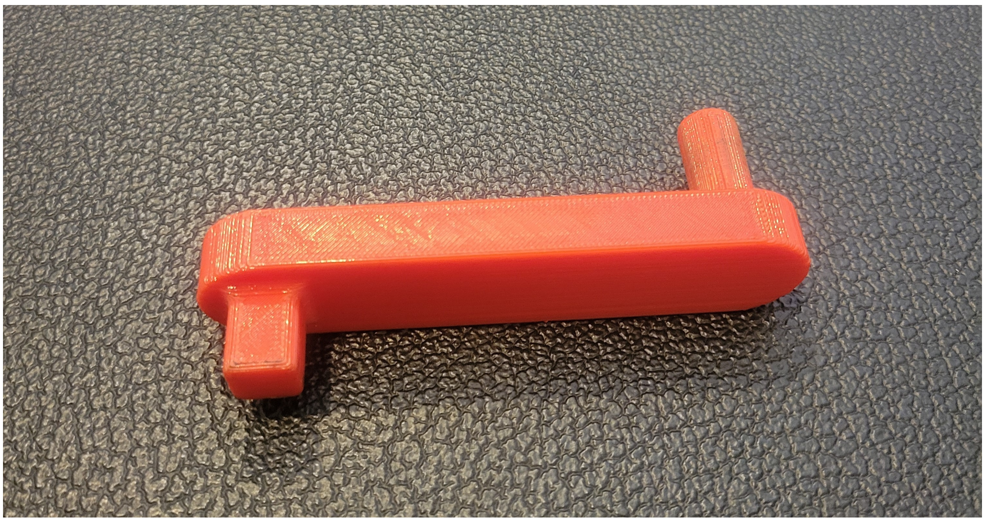

Figure 17. The 3D-printed handle.

4. Operating Instructions

To test a cam and collect data, the desired follower and cam are chosen. For this research study, the point follower is selected. The point follower is inserted up through the linear bearing in the dial indicator holder until it contacts the touch probe of the dial indicator shown in Figure 15. While lifting up on the follower, attach a cam to the rotating shaft using the thumb screw provided. Lower the follower onto the surface of the cam. Rotate the cam while reading the dial indicator to find the lowest point of the cam. This will be the starting point for data collection. Verify that the starting point aligns with the extruded boss on the face of the cam representing the theoretical lowest point. Once the starting point has been established, rotate the outer collar of the angle gauge shown in Figure 16 to zero the gauge. It is important to note that the rotating shaft must not move while zeroing the angle gauge, otherwise the data can be shifted in relation to angle. The dial indicator is then zeroed out by loosening the four screws holding the assembly into the frame shown in Figure 15. Slide the indicator holder down until both dials on the indicator (hundredths and thousandths) are within 0.010″ of the 0 mark. Tighten the four screws securing the fixture in place. Loosen the thumb screw on the dial indicator at the 3 O'clock position. Rotate the face of the indicator until the 0 mark aligns with the needle on the indicator. Tighten the thumb screw, securing the face from moving. The apparatus is now calibrated and ready to collect data.

To collect the data, the angle gauge is rotated to predefined intervals using the 3D-printed handle shown in Figure 17. For this experiment, intervals of 5 degrees are chosen. At each interval, the lift on the dial indicator is recorded. This process is repeated until the angle gauge is back to 0 degrees. At 5 degree intervals, 72 data points are recorded for each cam tested. The cam can then be removed and another attached to be measured. Since all of the cams are designed with the same maximum and minimum lift criteria, the dial indicator should not have to be re-calibrated if the same follower is used.

5. Validation

To validate the accuracy of the cam measuring machine, two reference cams were precision machined out of 6061-T6 aluminum. The cams were circles with outer diameters of 2 inches and 4 inches. The reference cams had run-out of plus or minus 0.0005″ measured using standard measuring equipment (SME). The dial indicator used in this machine has a resolution of 0.001″. Therefore, values between thousandths indications such as 0.0015″ were rounded up to the nearest thousandth. The reference cams were mounted to the measuring machine, and measurements were taken at 10 degree intervals.

After the cam measuring apparatus was validated, the displacements for the five 3D-printed cam profiles were measured at 5-degree angle intervals. The displacements were then plotted versus the angle. Velocity and acceleration data were calculated using numerical methods based on the displacement data, where the derivatives were replaced with discrete changes between data points.

5.1. Reference Cams

To measure the reference cams, the lowest point on the cam was selected as the starting point. Since the cams are circles with multiple values indicating 0.000″, the data were shifted between reference cams based on the starting point selected. However, the total indicated run-out graph depicts the same trend in displacement, as displayed in Figure 18. This indicates that there was a total run-out of 0.002″ in the shaft of the machine. Based on the reference cam data and analysis, plus or minus 0.001″ of error in the 3D-printed cam data collected can be attributed to the apparatus.

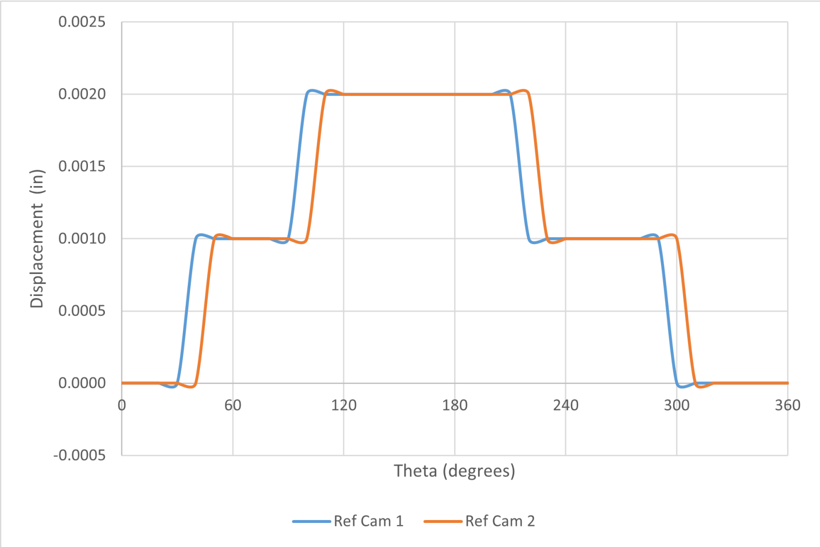

Figure 18. Reference cam total indicated run-out.

5.2. Double Dwell

The double-dwell cam was characterized by two distinct dwell phases: one at its apex and another at its nadir, each spanning 90 degrees. Despite its effective operation, the cam encountered intermittent sticking at the transition points during elevation and descent. To address this, fillets were integrated into the design to facilitate smoother transitions, albeit with a slight modification to the displacement curve. This adjustment resulted in a marginal discrepancy between the theoretical values and the experimental results. Figure 19 displays the displacement versus the angle rotated with the experimental data, depicted in orange, closely following the theoretical data, shown in blue. The average percentage error of the displacement values was 2.3%. Figure 20 displays the velocity versus the angle with data points that had a larger deviation from the theoretical path at the transitions compared to the dwells. Figure 21 displays the acceleration versus the angle with the most noticeable error occurring during the linear transitions between the dwells. This discrepancy is primarily attributed to the fact that the small error during displacement is squared in velocity and then again in acceleration.

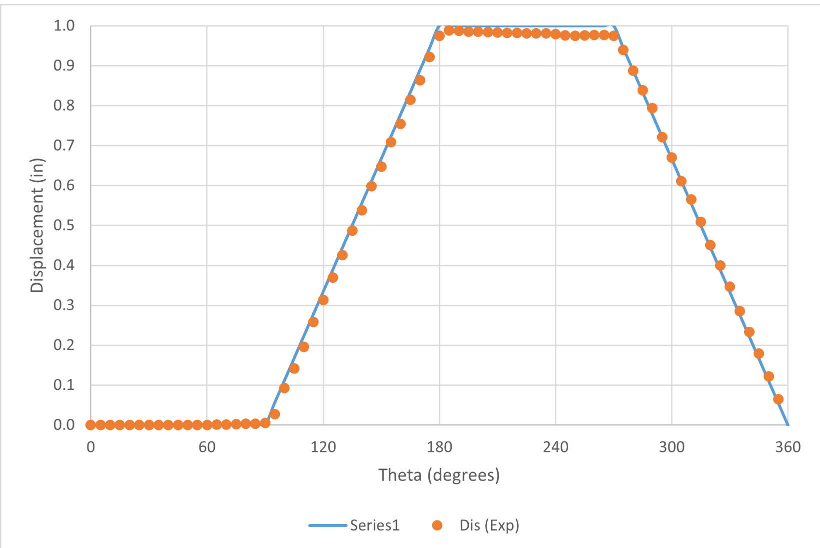

Figure 19. Double-dwell displacement.

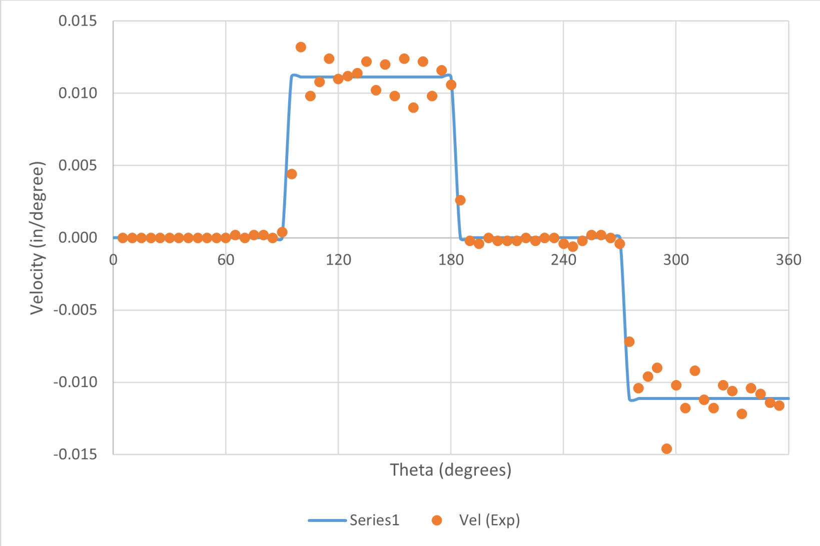

Figure 20. Double-dwell velocity.

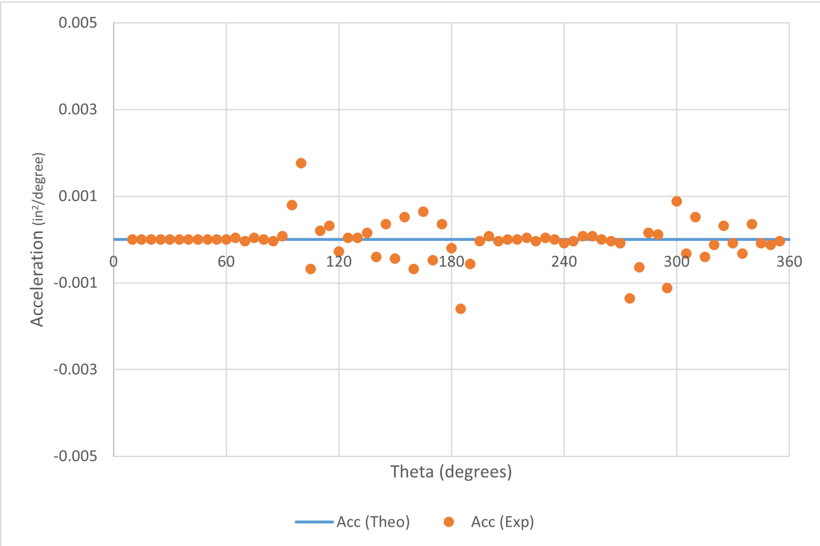

Figure 21. Double-dwell acceleration.

5.3. Linear Motion (Up Only)

The linear motion (up only) cam was characterized by the displacement of the follower and angle rotated to be directly proportional. This relationship was observed to exhibit a linear increase, represented by a straight line, as demonstrated in Figure 22. The average percentage error of the displacement values was 1.7%. In Figure 23, velocity versus rotation, the experimental data follow the theoretical curve with minor deviations. Since this cam exhibits a linear motion with constant velocity, the acceleration was zero. Figure 24 displays the experimental results for acceleration.

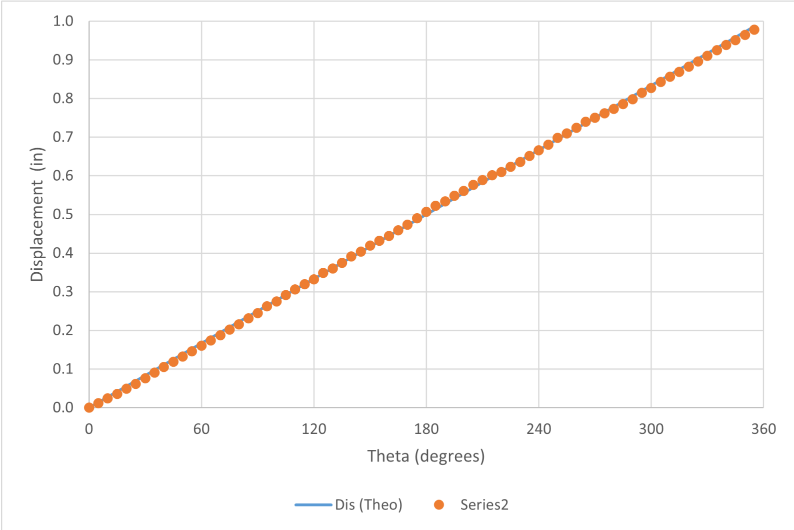

Figure 22. Linear motion (up only) displacement.

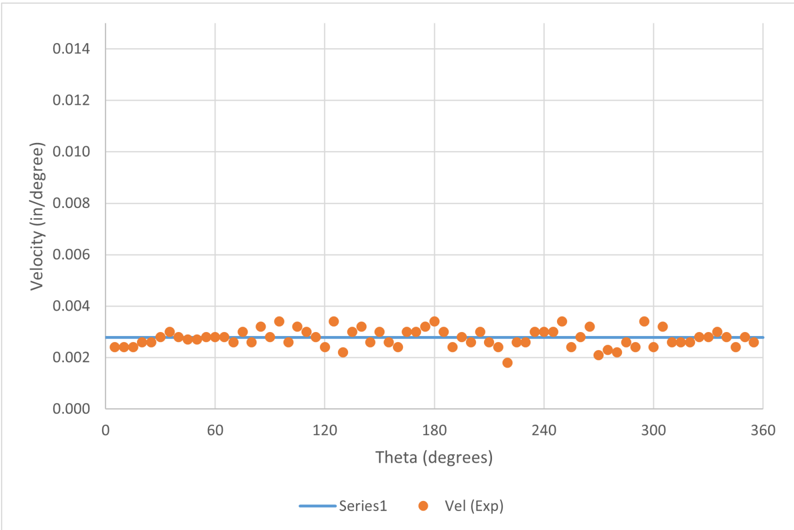

Figure 23. Linear motion (up only) velocity.

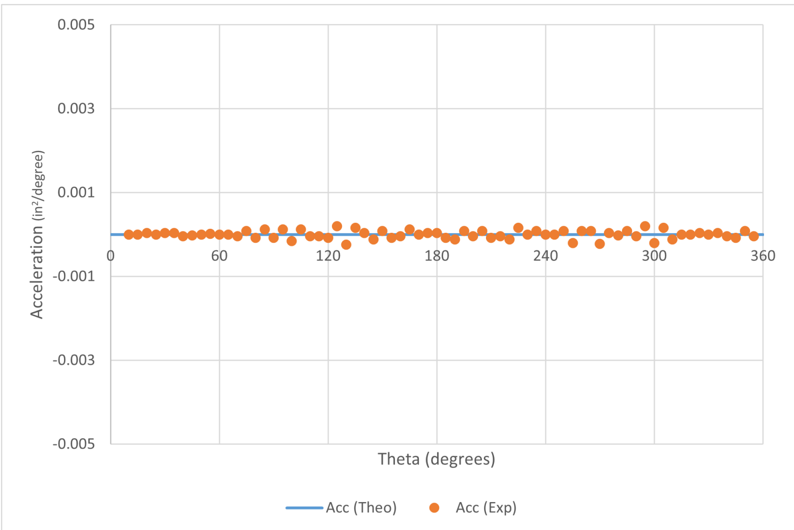

Figure 24. Linear motion (up only) acceleration.

5.4. Linear Motion (Up and Down)

Similar to the linear motion (up only) cam, the linear motion (up and down) cam increases displacement for 180 degrees and then decreases displacement for the remaining 180 degrees. Its velocity remains constant, positive for the first half and negative for the second half. Figure 25 demonstrates the steady increase and decrease of displacement versus the degree rotated. The average percentage error of the displacement values was 7.5%. Figure 26 displays the constant velocity for each increasing and decreasing displacement. Figure 27 displays the acceleration, which is zero.

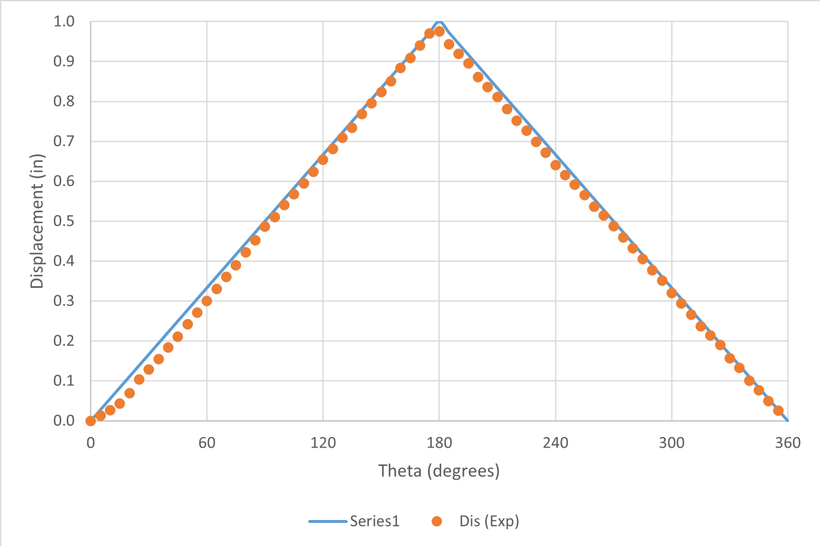

Figure 25. Linear motion (up and down) displacement.

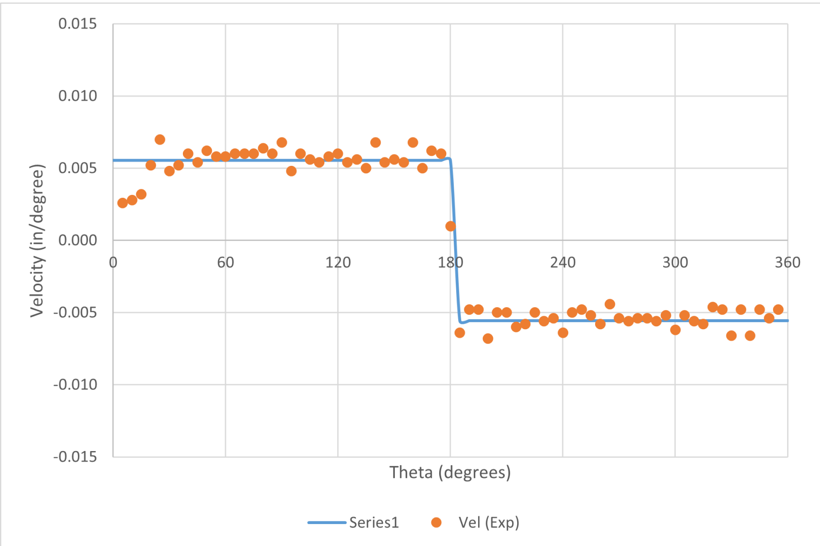

Figure 26. Linear motion (up and down) velocity.

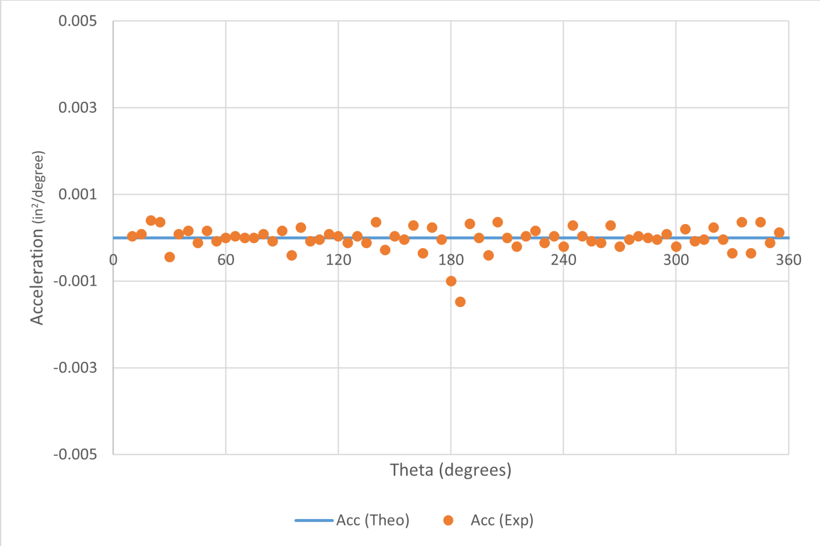

Figure 27. Linear motion (up and down) acceleration.

5.5. Harmonic Motion

The harmonic motion cam profile was designed to follow a harmonic motion curve, which utilizes cosine waveforms. In Figure 28, the displacement versus angle displays a general trend that follows the theoretical model. The average percentage error of the displacement values was 2.1%. Figure 29, velocity versus angle, also follows a trend with minor inaccuracies at extremes. Figure 30 displays the acceleration.

5.6. Cycloidal Motion

The cycloidal motion cam vaguely resembles the harmonic motion cam but is slightly flattened at the apex and nadir. Figure 31, the displacement versus angle, displays a general trend that follows the theoretical model. The average percentage error of the displacement values was 7.5%. Figure 32, velocity versus angle, also follows the expected trend with slight inaccuracies at the maximum and minimum velocities. Figure 33 displays the acceleration.

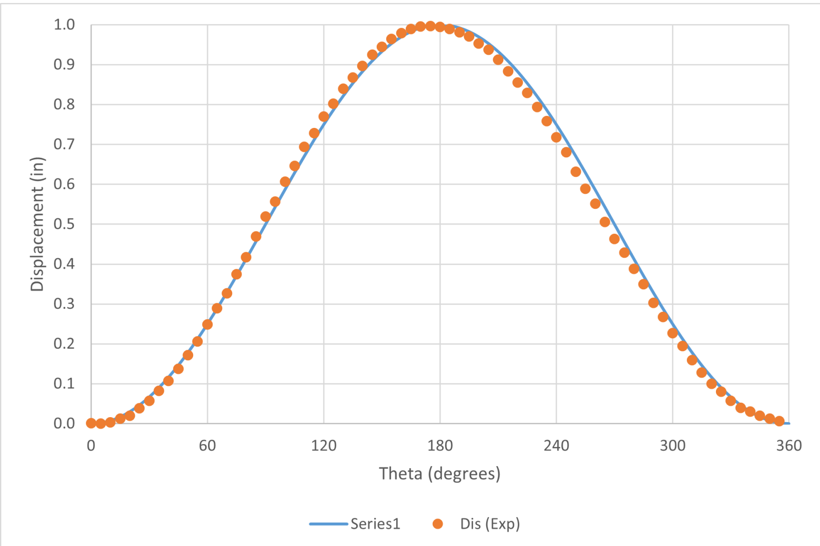

Figure 28. Harmonic motion displacement.

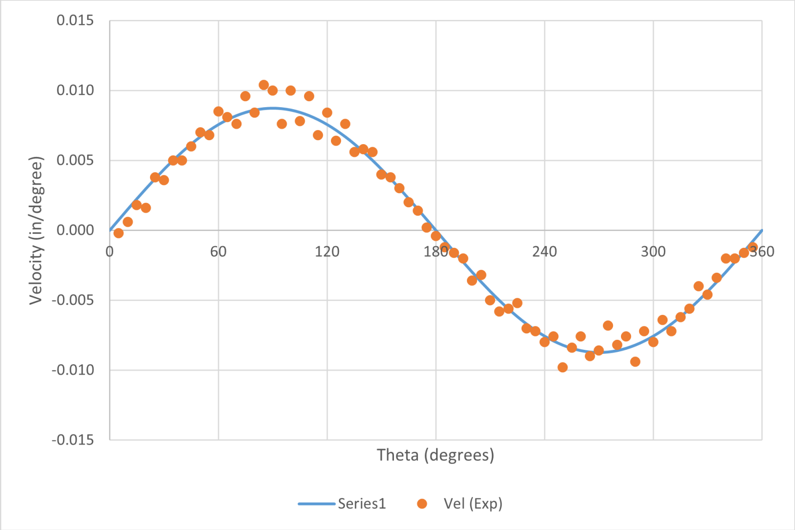

Figure 29. Harmonic motion velocity.

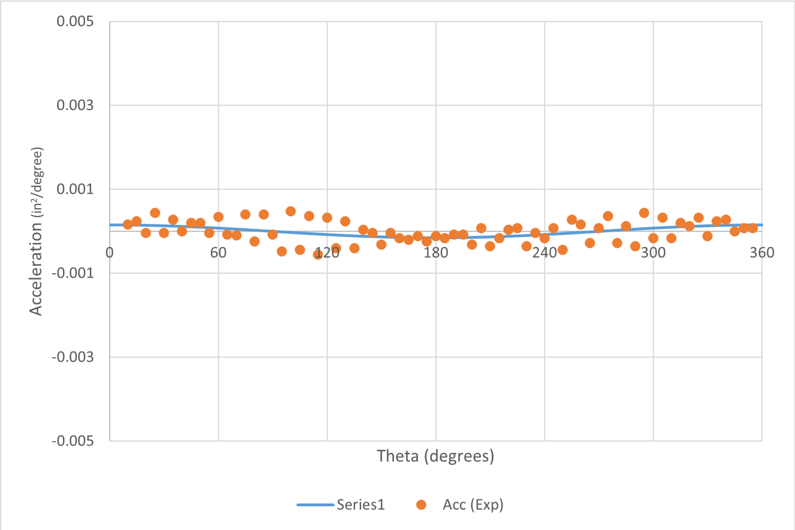

Figure 30. Harmonic motion acceleration.

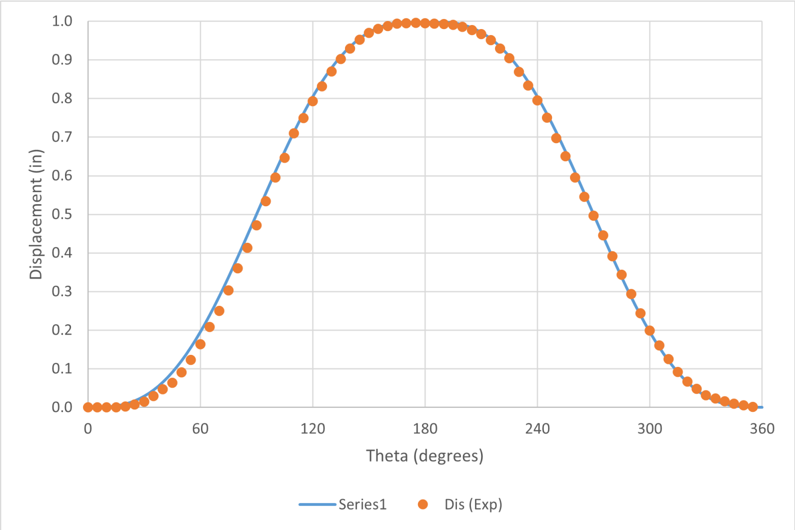

Figure 31. Cycloidal motion displacement.

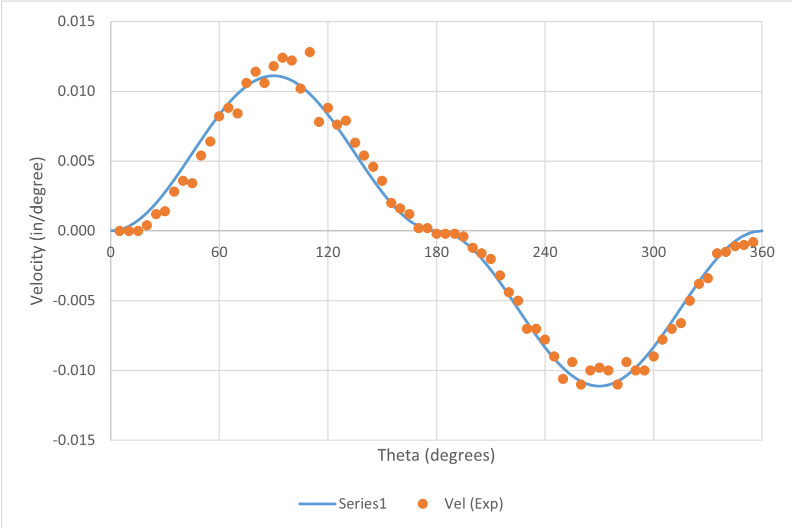

Figure 32. Cycloidal motion velocity.

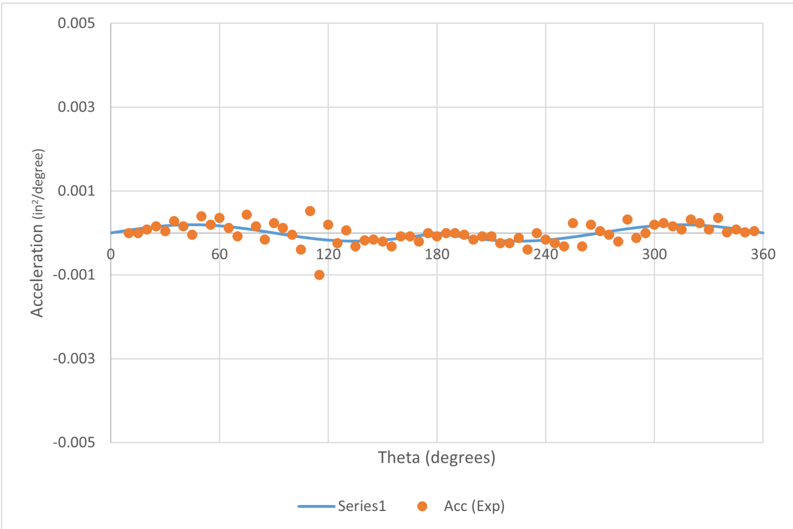

Figure 33. Cycloidal motion acceleration.

6. Conclusions

This research study aimed to construct a laboratory cam profile measuring machine and evaluate cam profiles, effectively demonstrating the principles of cam operation and facilitating the design of various cam shapes. The project successfully achieved these objectives, presenting a comprehensive laboratory apparatus that visually and quantitatively demonstrates the mechanics of cam action.

The cam machine, equipped with various cam profiles, accurately measured displacement through a calibrated system, utilizing an angle gauge and a linear dial indicator. The translation of theoretical displacement curves into physical cam shapes was achieved by modeling in SolidWorks, and translating that computer model to a physical model by 3D printing. Subsequent testing confirmed the integrity of these cams to their intended displacement profiles, with minimal discrepancies measured. The laboratory cam measuring machine's accuracy was validated using reference cams. Based on the total indicated run-out of the machine, 0.2% of the error in the 3D-printed cams can be attributed to the machine itself. The 3D-printed cams performed well, exhibiting an average percentage error of the displacement values of 2.3% for the double dwell, 1.7% for the linear motion (up only), 7.5% for the linear motion (up and down), 2.1% for harmonic motion, and 7.5% for cycloidal motion.

The velocities were calculated from the displacement data which generally followed the theoretical curves. A source of error was the backlash in the coupling between the angle gauge and the rotating shaft that caused the cam to rotate slightly, causing the displacement to be measured at an unintended angle. This compounded the error in both velocity and acceleration curves. This highlighted the potential for the replacement of the analog measurement device with a digital measurement device to ensure more accurate and precise measurements. A stepper motor can be added in replacement of the angle gauge to measure at a tighter tolerance or to measure displacement as a function of time. Further modifications, such as offsetting the follower or using different types of followers, may be implemented to expand the capabilities of this machine. The calculations of velocity and acceleration for non-linear curves indicated a need for finer increments during data recording, suggesting a shift from 5-degree to 1-degree or smaller increments to improve accuracy. Improvements in reducing error in displacement curves can be made by acquiring a more precise 3D printer or producing higher-quality prints. It is crucial to highlight that the method of obtaining velocity and acceleration of follower motion using numerical differentiation amplifies any inaccuracies in measurement or data collection, thus adding to the error.

In conclusion, a cam profile measuring machine was designed and fabricated for a laboratory setting. A variety of 3D-printed cams were tested and compared to theoretical values for displacement with minimal errors. This machine not only serves as a powerful tool offering insight into the functionality of cams but also allows for further innovations in cam design and applications.

Author Contributions: Conceptualization, E.B.III; methodology, E.B.III, A.A., J.P. and J.S.; validation, A.A., J.P. and J.S.; formal analysis, E.B.III; investigation, J.P. and J.S.; resources, E.B.III; data curation, A.A.; writing—original draft preparation, A.A., J.P. and J.S.; writing—review and editing, A.A., J.P. and J.S.; visualization, E.B.III; supervision, E.B.III; project administration, E.B.III; funding acquisition, E.B.III. All authors have read and agreed to the published version of the manuscript.

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Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

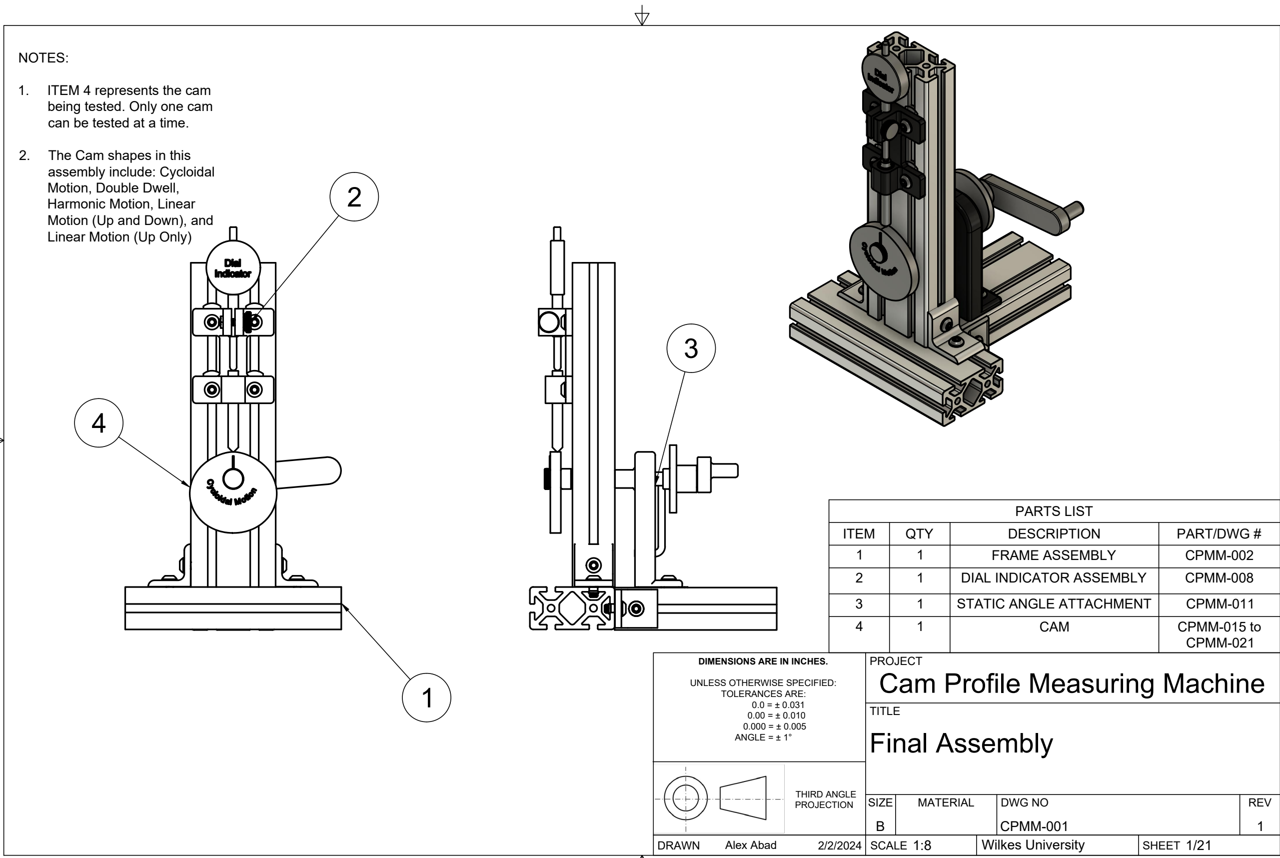
Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflict of interest.

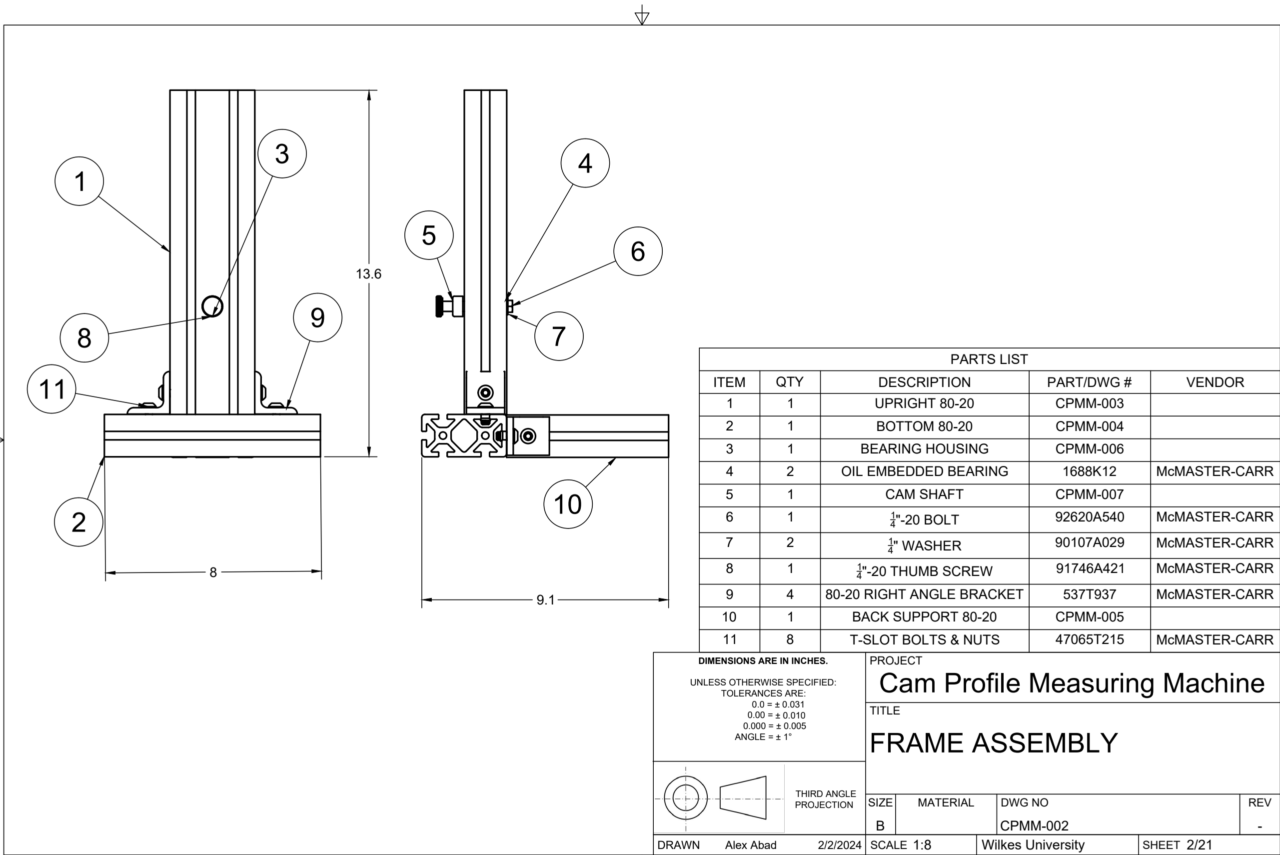
Abbreviations

θ	Degrees
h	Maximum Follower Lift
$R_{\max}$	Maximum Radius of the Cam
$R_{\min}$	Minimum Radius of the Cam
d	Displacement
v	Velocity
a	Acceleration

Appendix A. Drawing Packet



PARTS LIST			
ITEM	QTY	DESCRIPTION	PART/DWG #
1	1	FRAME ASSEMBLY	CPMM-002
2	1	DIAL INDICATOR ASSEMBLY	CPMM-008
3	1	STATIC ANGLE ATTACHMENT	CPMM-011
4	1	CAM	CPMM-015 to CPMM-021



PARTS LIST				
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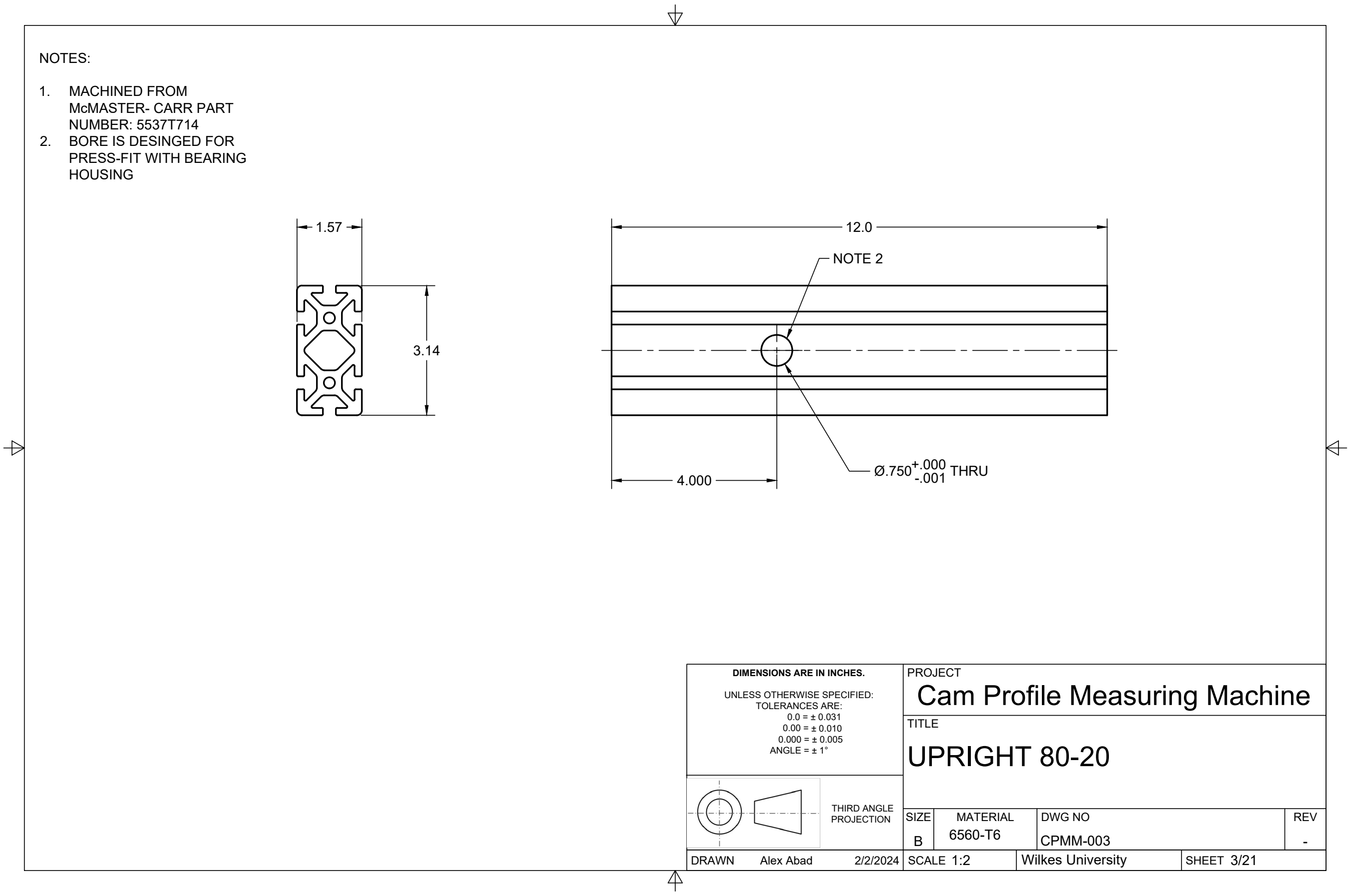
1	1	UPRIGHT 80-20	CPMM-003	
2	1	BOTTOM 80-20	CPMM-004	
3	1	BEARING HOUSING	CPMM-006	
4	2	OIL EMBEDDED BEARING	1688K12	McMASTER-CARR
5	1	CAM SHAFT	CPMM-007	
6	1	$\frac{1}{4}$"-20 BOLT	92620A540	McMASTER-CARR
7	2	$\frac{1}{4}$" WASHER	90107A029	McMASTER-CARR
8	1	$\frac{1}{4}$"-20 THUMB SCREW	91746A421	McMASTER-CARR
9	4	80-20 RIGHT ANGLE BRACKET	537T937	McMASTER-CARR
10	1	BACK SUPPORT 80-20	CPMM-005	
11	8	T-SLOT BOLTS & NUTS	47065T215	McMASTER-CARR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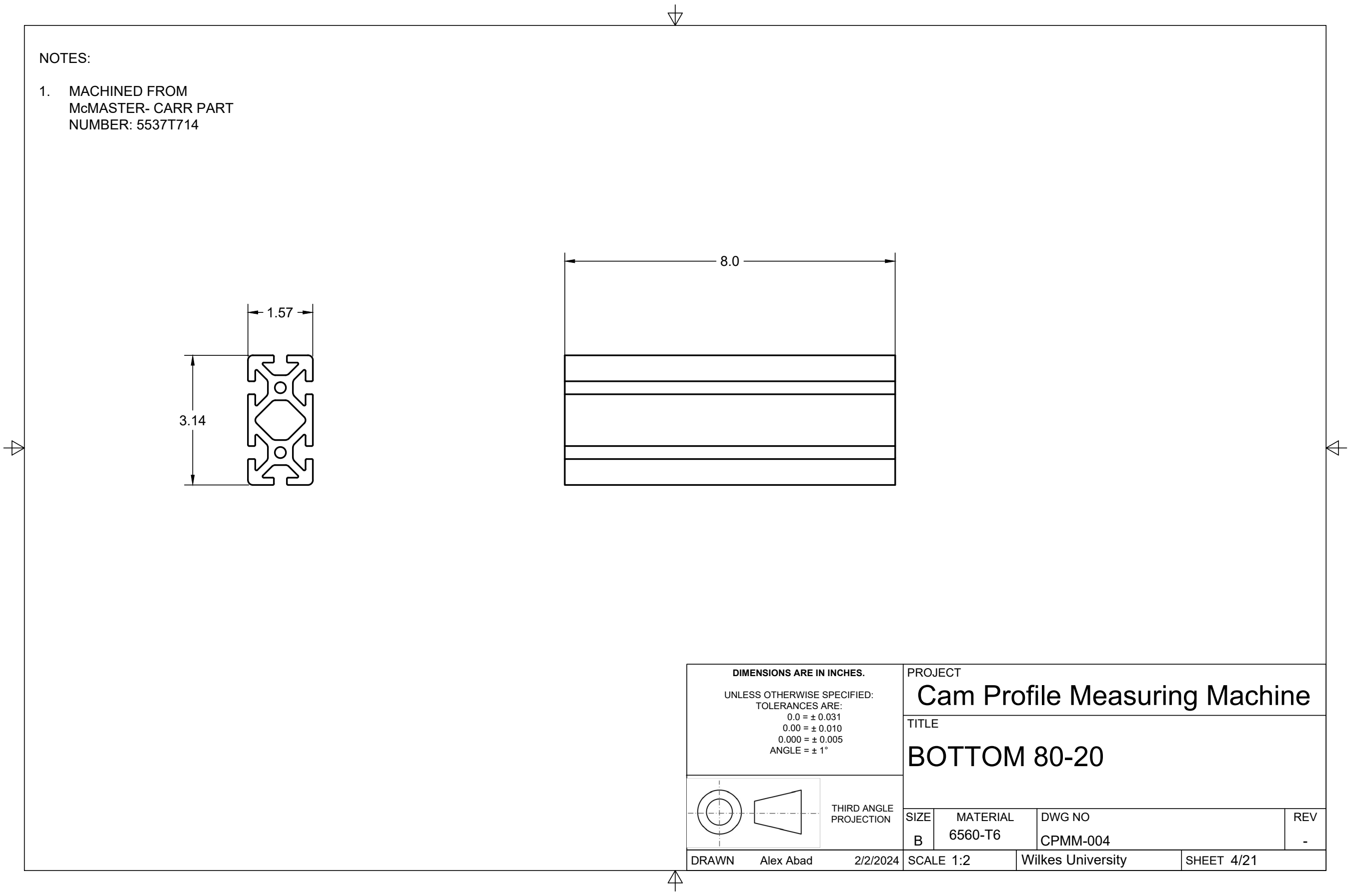
3.14
12.0
NOTE 2
4.000
Ø.750 +.000 -.001 THRU
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE: 0.0 = ± 0.031 0.00 = ± 0.010 0.000 = ± 0.005 ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
UPRIGHT 80-20
SIZE B
MATERIAL 6560-T6
DWG NO CPMM-003
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:2
Wilkes University
SHEET 3/21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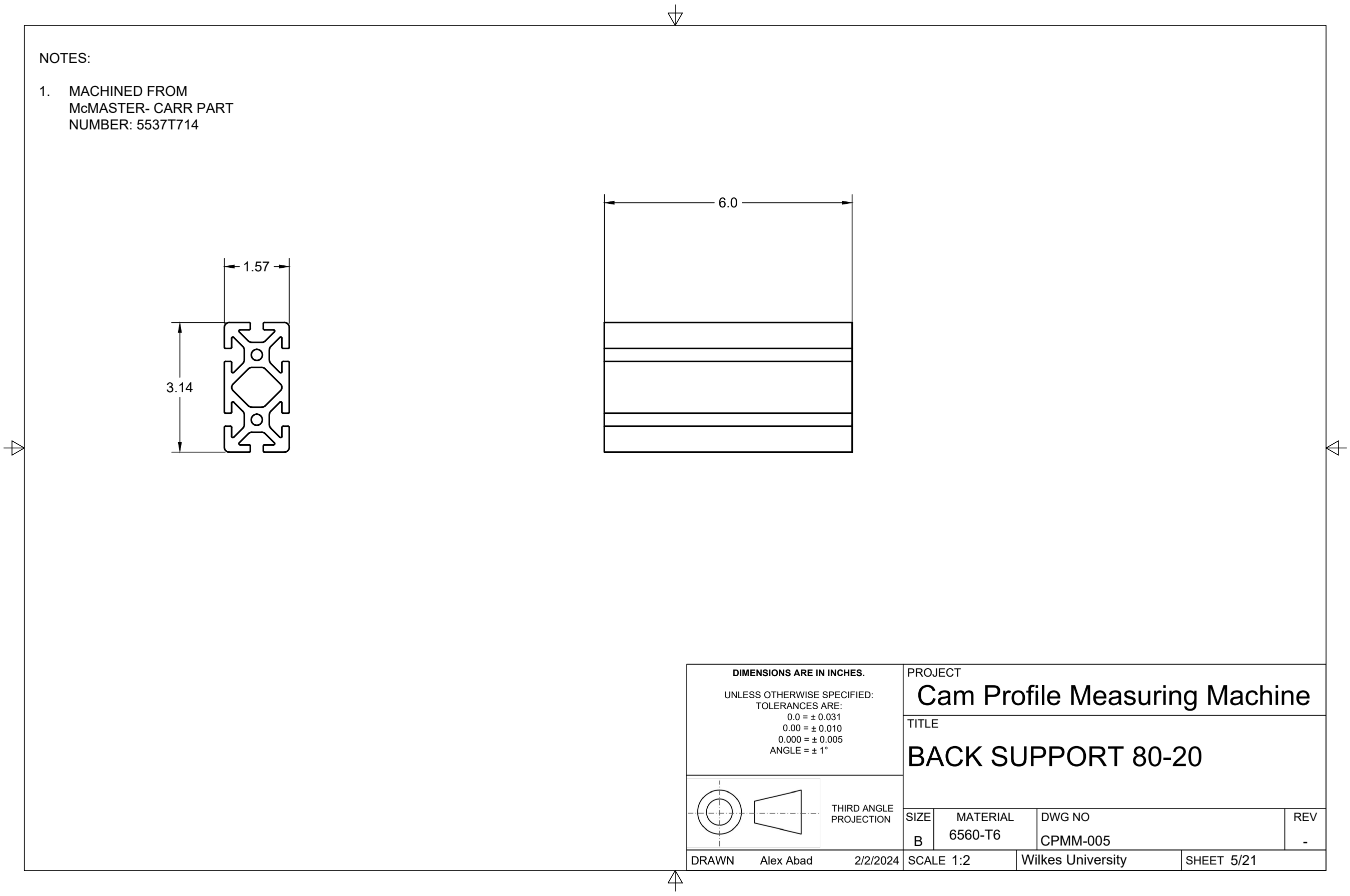
3.14
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE: 0.0 = ± 0.031 0.00 = ± 0.010 0.000 = ± 0.005 ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
BOTTOM 80-20
SIZE B
MATERIAL 6560-T6
DWG NO CPMM-004
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:2
Wilkes University
SHEET 4/21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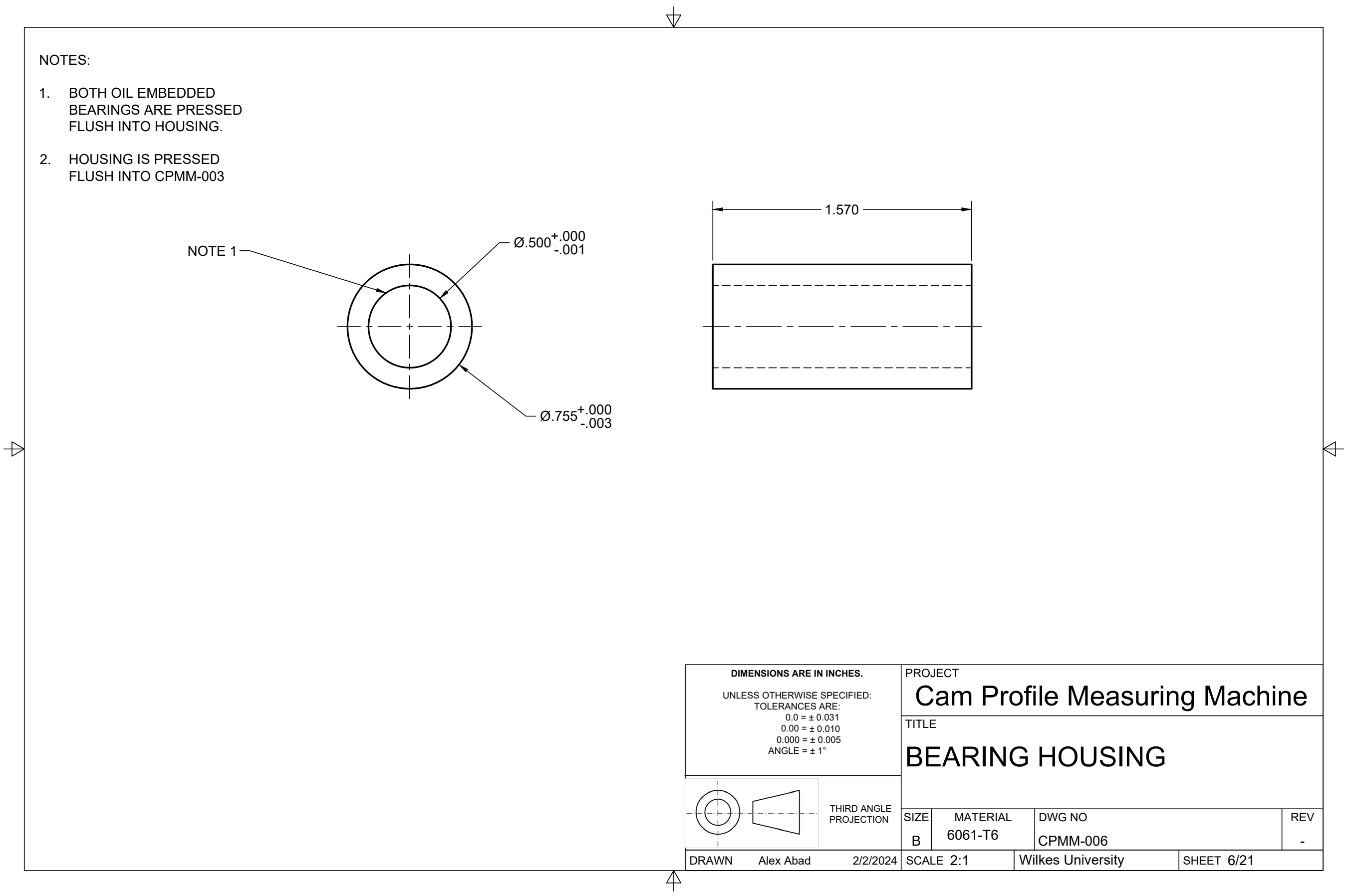
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
BACK SUPPORT 80-20
SIZE B
MATERIAL 6560-T6
DWG NO CPMM-005
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:2
Wilkes University
SHEET 5/21


NOTES:
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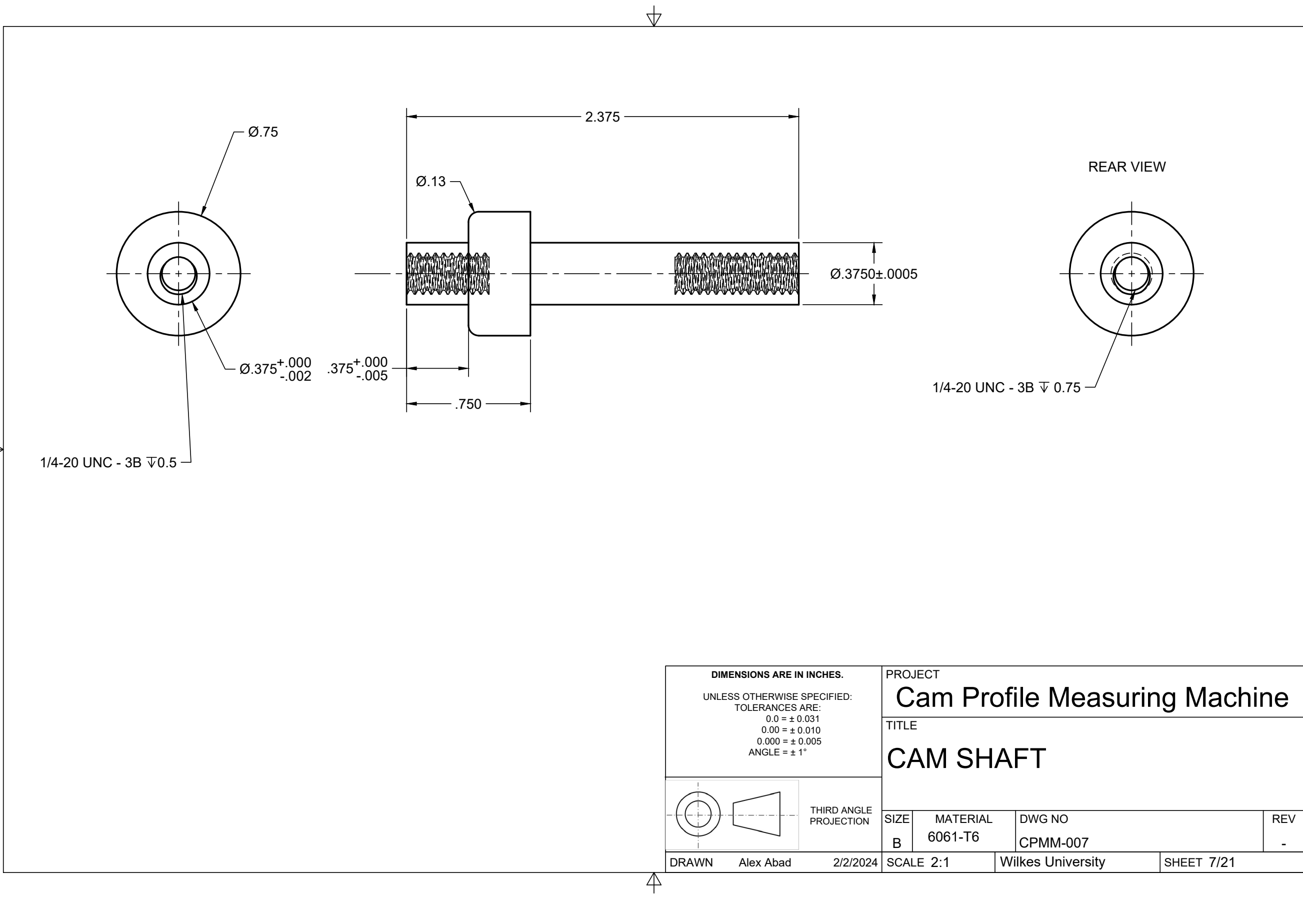
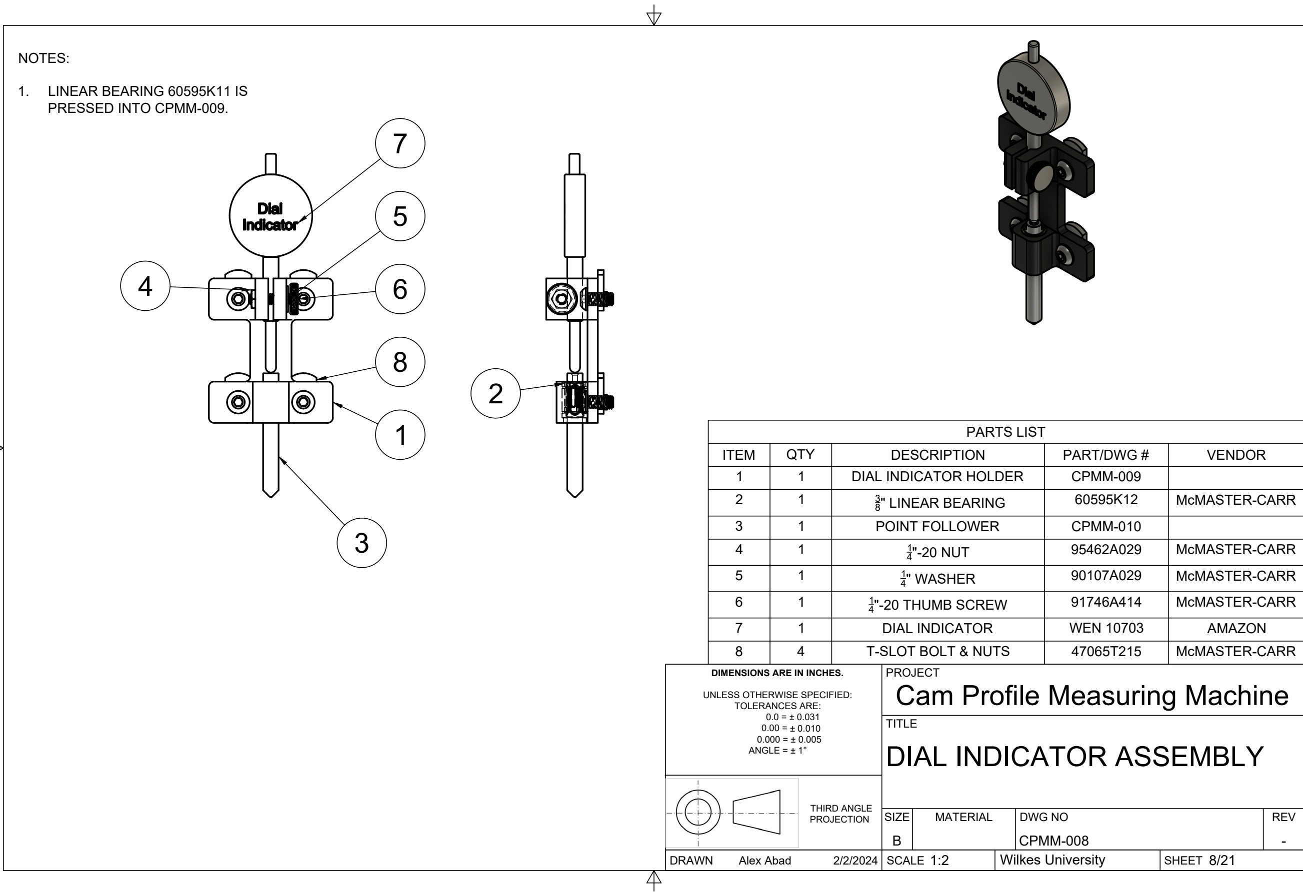
2. HOUSING IS PRESSED FLUSH INTO CPMM-003
NOTE 1
Ø.500 +.000 -.001
Ø.755 +.000 -.003
1.570
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
BEARING HOUSING
SIZE B
MATERIAL 6061-T6
DWG NO CPMM-006
REV -
DRAWN Alex Abad 2/2/2024
SCALE 2:1
Wilkes University
SHEET 6/21





PARTS LIST

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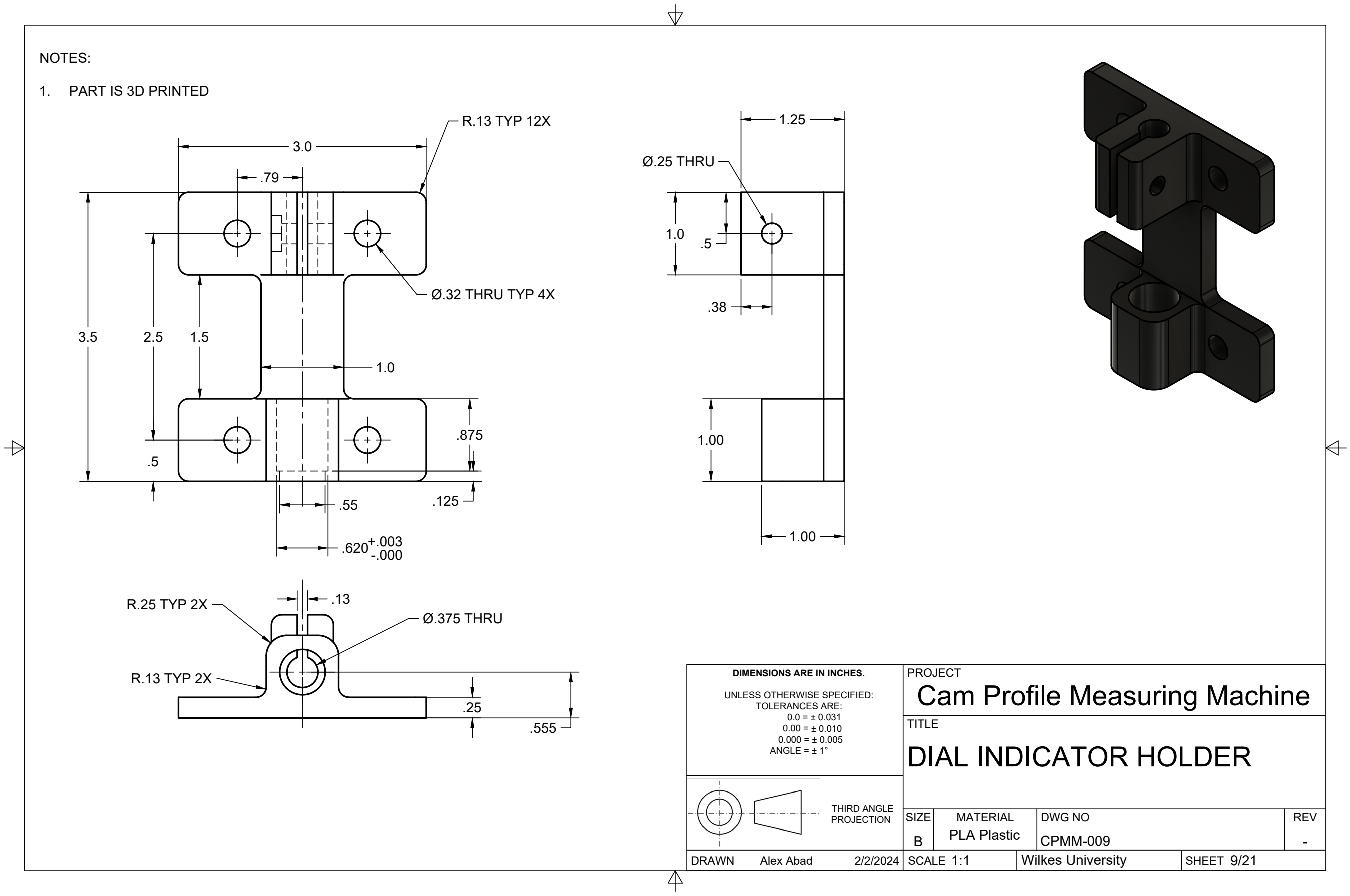
2	1	3/8" LINEAR BEARING	60595K12	McMASTER-CARR
3	1	POINT FOLLOWER	CPMM-010	
4	1	1/4"-20 NUT	95462A029	McMASTER-CARR
5	1	1/4" WASHER	90107A029	McMASTER-CARR
6	1	1/4"-20 THUMB SCREW	91746A414	McMASTER-CARR
7	1	DIAL INDICATOR	WEN 10703	AMAZON
8	4	T-SLOT BOLT & NUTS	47065T215	McMASTER-CARR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Ø.32 THRU TYP 4X
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2.5
1.5
1.0
.875
.5
.55
.125
.620 +.003 -.000
R.25 TYP 2X
.13
Ø.375 THRU
R.13 TYP 2X
.25
.555
1.25
Ø.25 THRU
1.0
.5
.38
1.00
1.00
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UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
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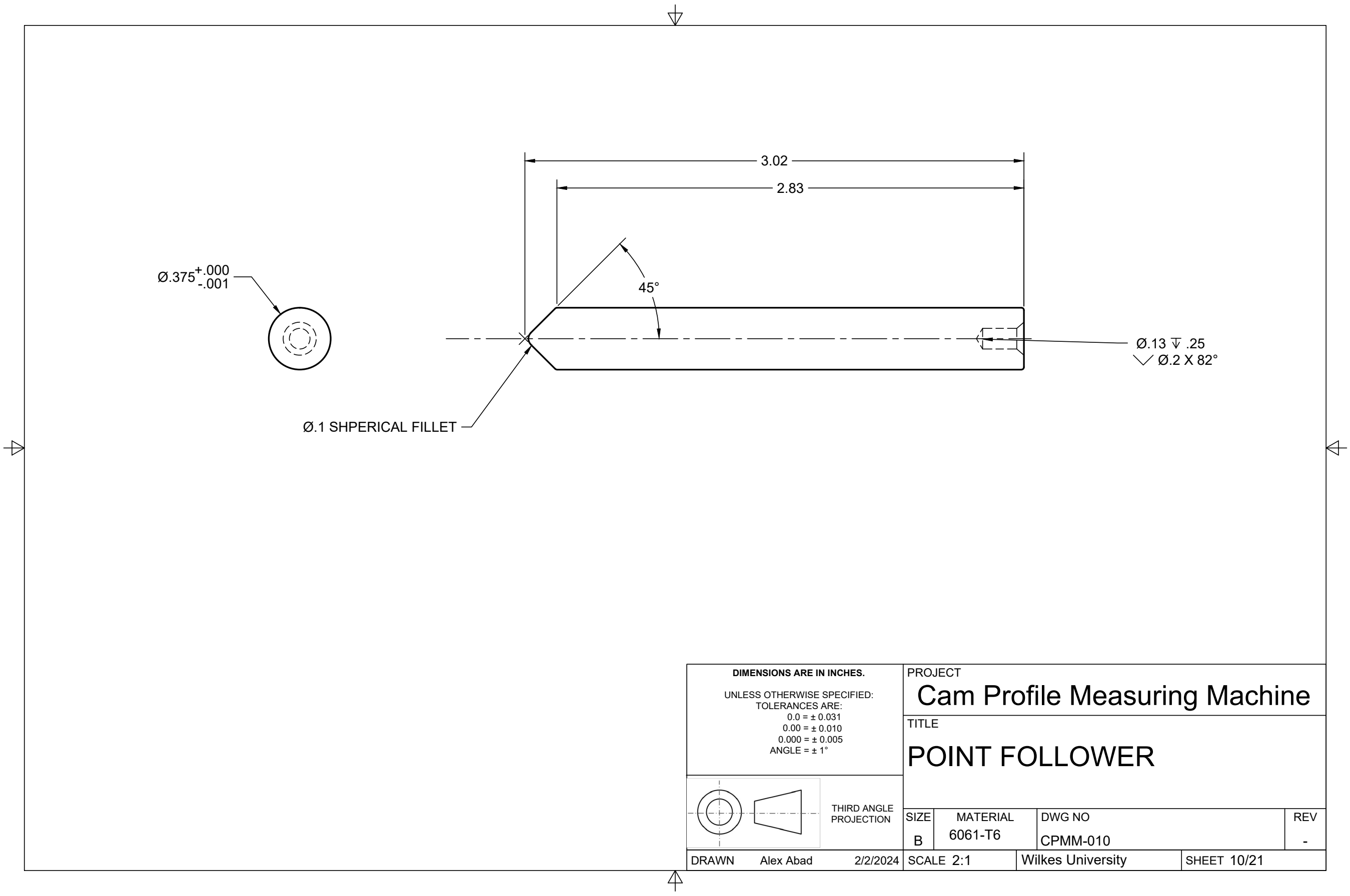
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
DIAL INDICATOR HOLDER
SIZE B
MATERIAL PLA Plastic
DWG NO CPMM-009
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:1
Wilkes University
SHEET 9/21


3.02
2.83
45°
Ø.375 +.000 -.001
Ø.13 ⩡ .25
⌵ Ø.2 X 82°
Ø.1 SHPERICAL FILLET
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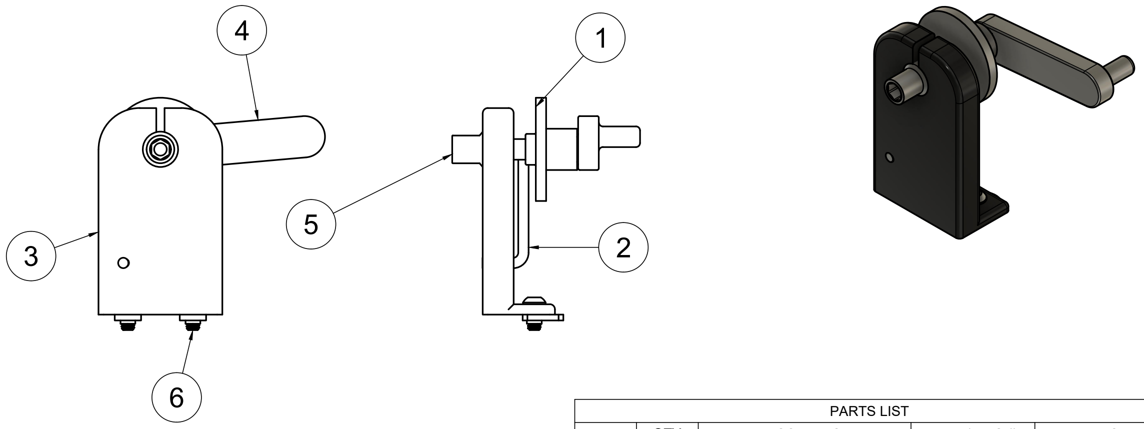
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
POINT FOLLOWER
SIZE B
MATERIAL 6061-T6
DWG NO CPMM-010
REV -
DRAWN Alex Abad 2/2/2024
SCALE 2:1
Wilkes University
SHEET 10/21



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ITEM	QTY	DESCRIPTION	PART/DWG #	VENDOR
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2	1	ANGLE GAUGE SUPPORT	CPMM-012	
3	1	ANGLE GAUGE HOLDER	CPMM-013	
4	1	CRANK HANDLE	CPMM-014	
5	1	$\frac{7}{16}$" SOCKET	5545A12	McMASTER-CARR
6	2	T-SLOT BOLTS & NUTS	47065T215	McMASTER-CARR

DIMENSIONS ARE IN INCHES.

UNLESS OTHERWISE SPECIFIED:
TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°

THIRD ANGLE PROJECTION

PROJECT: Cam Profile Measuring Machine

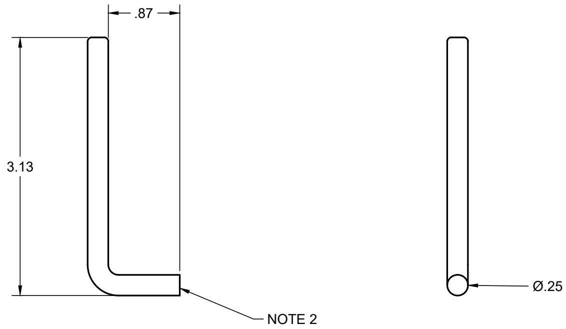
TITLE: STATIC ANGLE ATTACHMENT

SIZE	MATERIAL	DWG NO	REV
B		CPMM-011	-

DRAWN Alex Abad 2/2/2024 | SCALE 1:2 | Wilkes University | SHEET 11/21

NOTES:

1. PART IS SUPPLIED WITH SF0136
2. PART NEEDS TO BE MODIFIED TO FUNCTION PROPERLY



DIMENSIONS ARE IN INCHES.

UNLESS OTHERWISE SPECIFIED:
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0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°

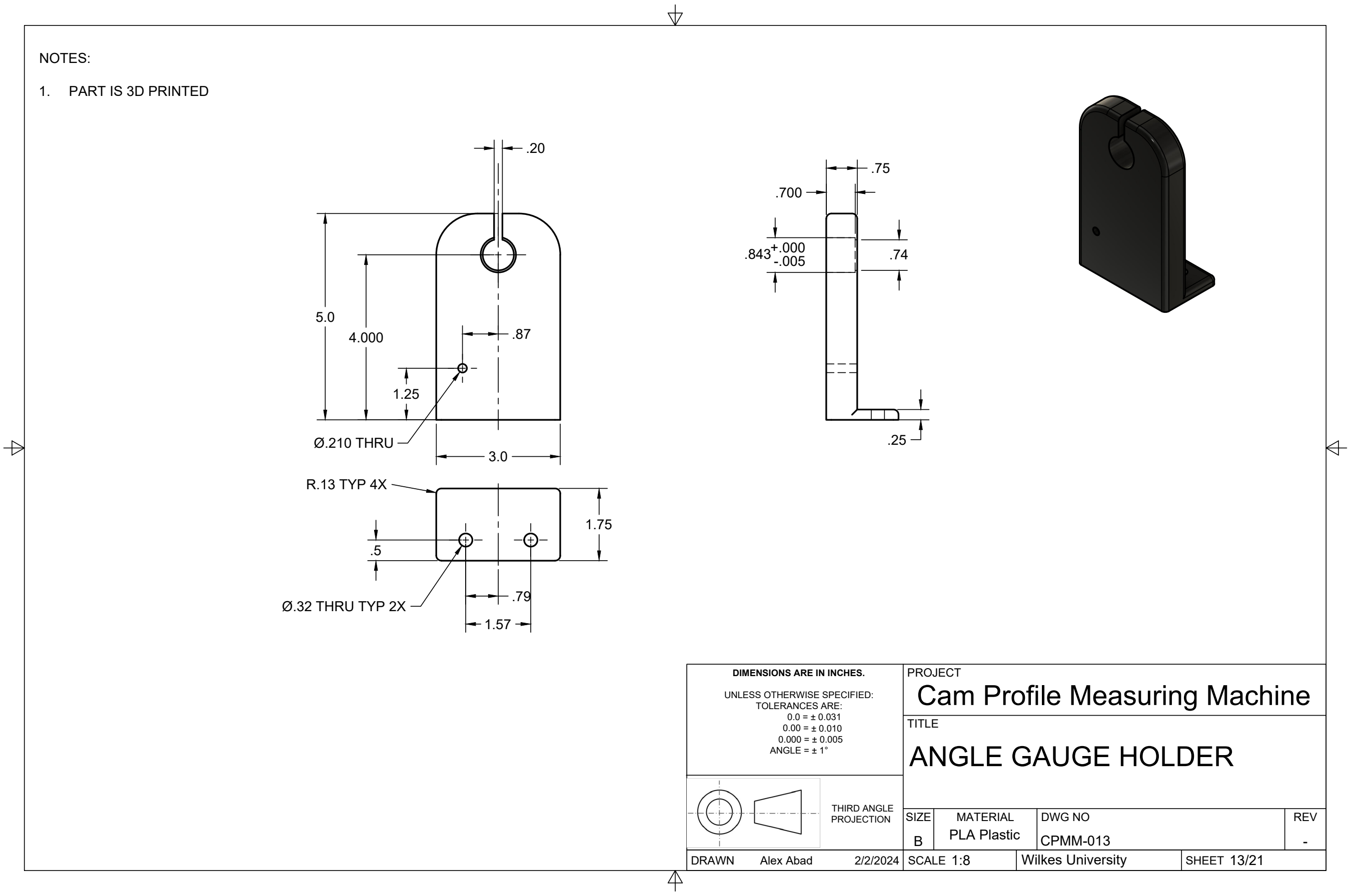
THIRD ANGLE PROJECTION

PROJECT: Cam Profile Measuring Machine

TITLE: ANGLE GAUGE SUPPORT

SIZE	MATERIAL	DWG NO	REV
B	STEEL	CPMM-012	-

DRAWN Alex Abad 2/2/2024 | SCALE 1:8 | Wilkes University | SHEET 12/21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4.000
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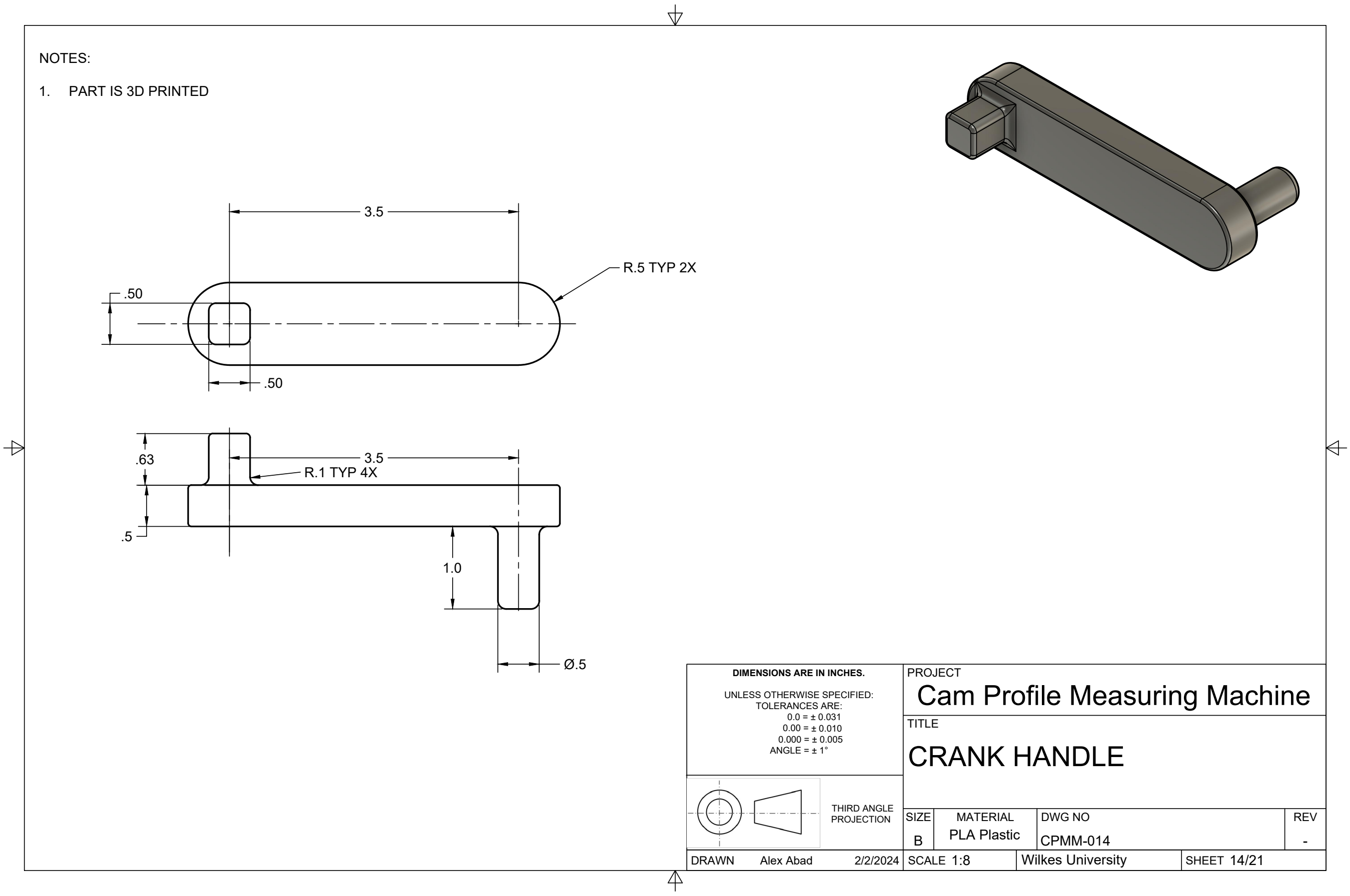
1.25
Ø.210 THRU
3.0
.25
R.13 TYP 4X
1.75
.5
Ø.32 THRU TYP 2X
.79
1.57
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED:
TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
ANGLE GAUGE HOLDER
SIZE B
MATERIAL PLA Plastic
DWG NO CPMM-013
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:8
Wilkes University
SHEET 13/21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3.5
R.1 TYP 4X
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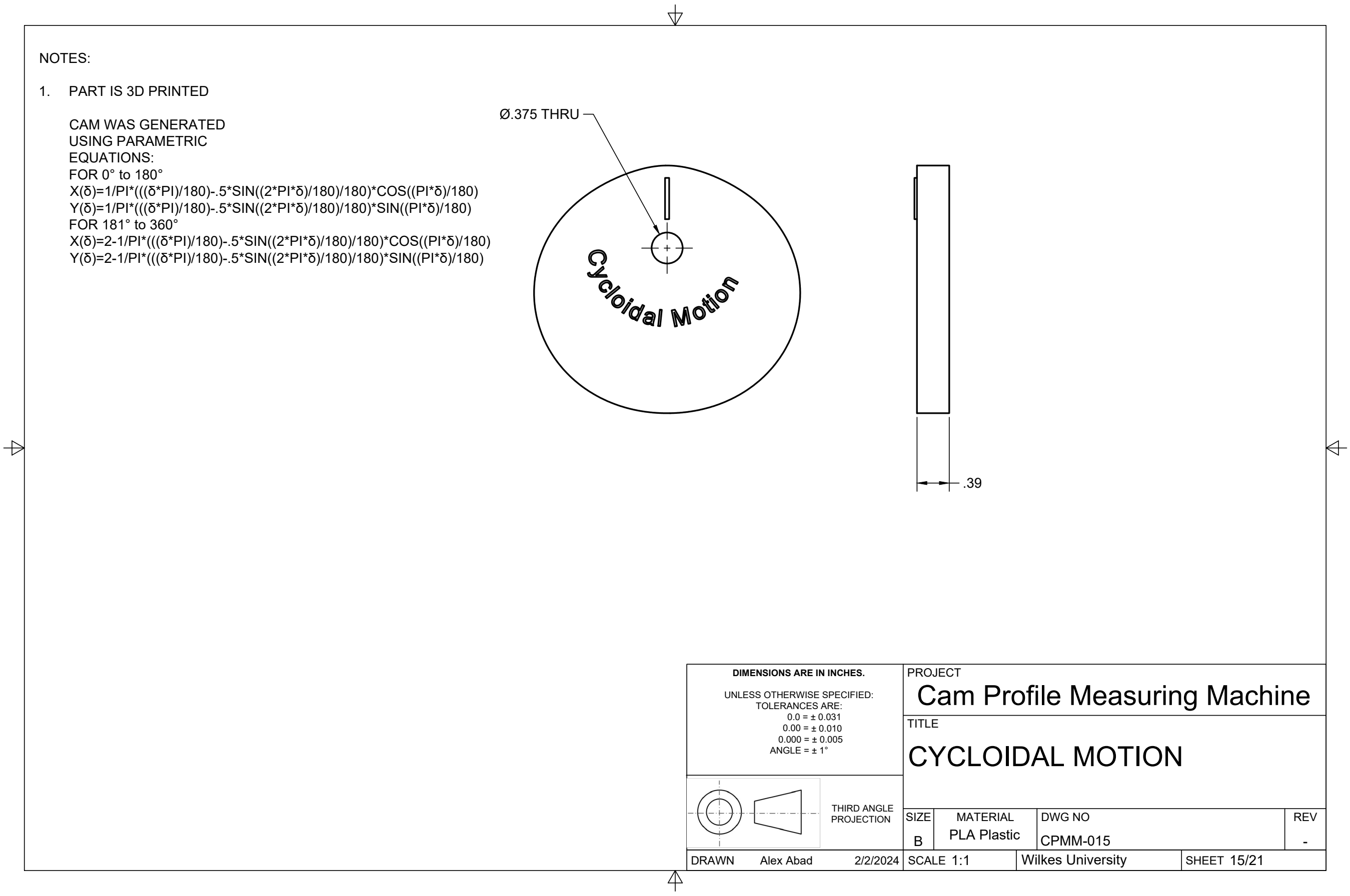
1.0
Ø.5
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED:
TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
CRANK HANDLE
SIZE B
MATERIAL PLA Plastic
DWG NO CPMM-014
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:8
Wilkes University
SHEET 14/21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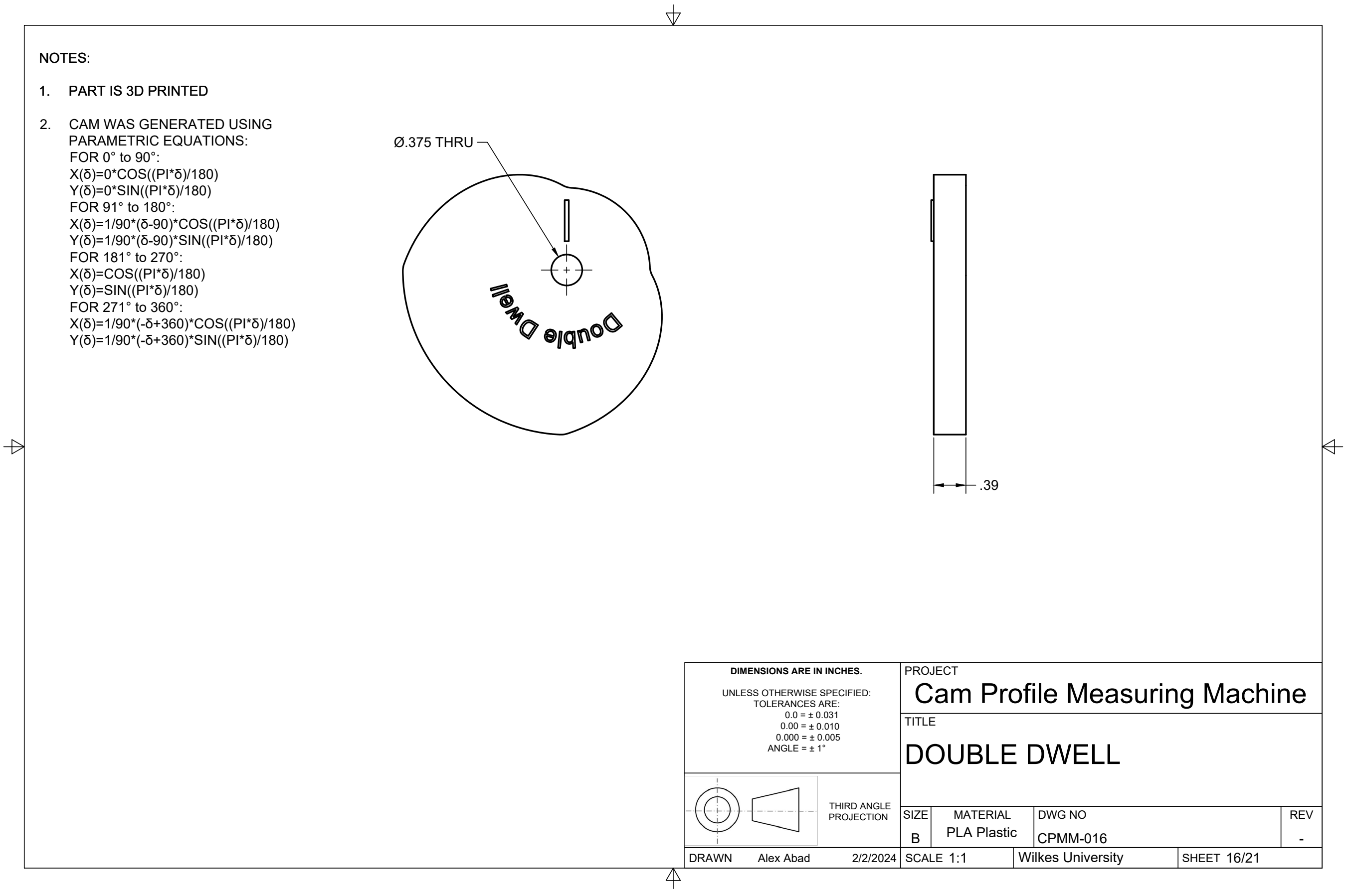
Y(δ)=1/PI*(((δ*PI)/180)-.5*SIN((2*PI*δ)/180)/180)*SIN((PI*δ)/180)
FOR 181° to 360°
X(δ)=2-1/PI*(((δ*PI)/180)-.5*SIN((2*PI*δ)/180)/180)*COS((PI*δ)/180)
Y(δ)=2-1/PI*(((δ*PI)/180)-.5*SIN((2*PI*δ)/180)/180)*SIN((PI*δ)/180)
Ø.375 THRU
Cycloidal Motion
.39
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
CYCLOIDAL MOTION
SIZE B
MATERIAL PLA Plastic
DWG NO CPMM-015
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:1
Wilkes University
SHEET 15/21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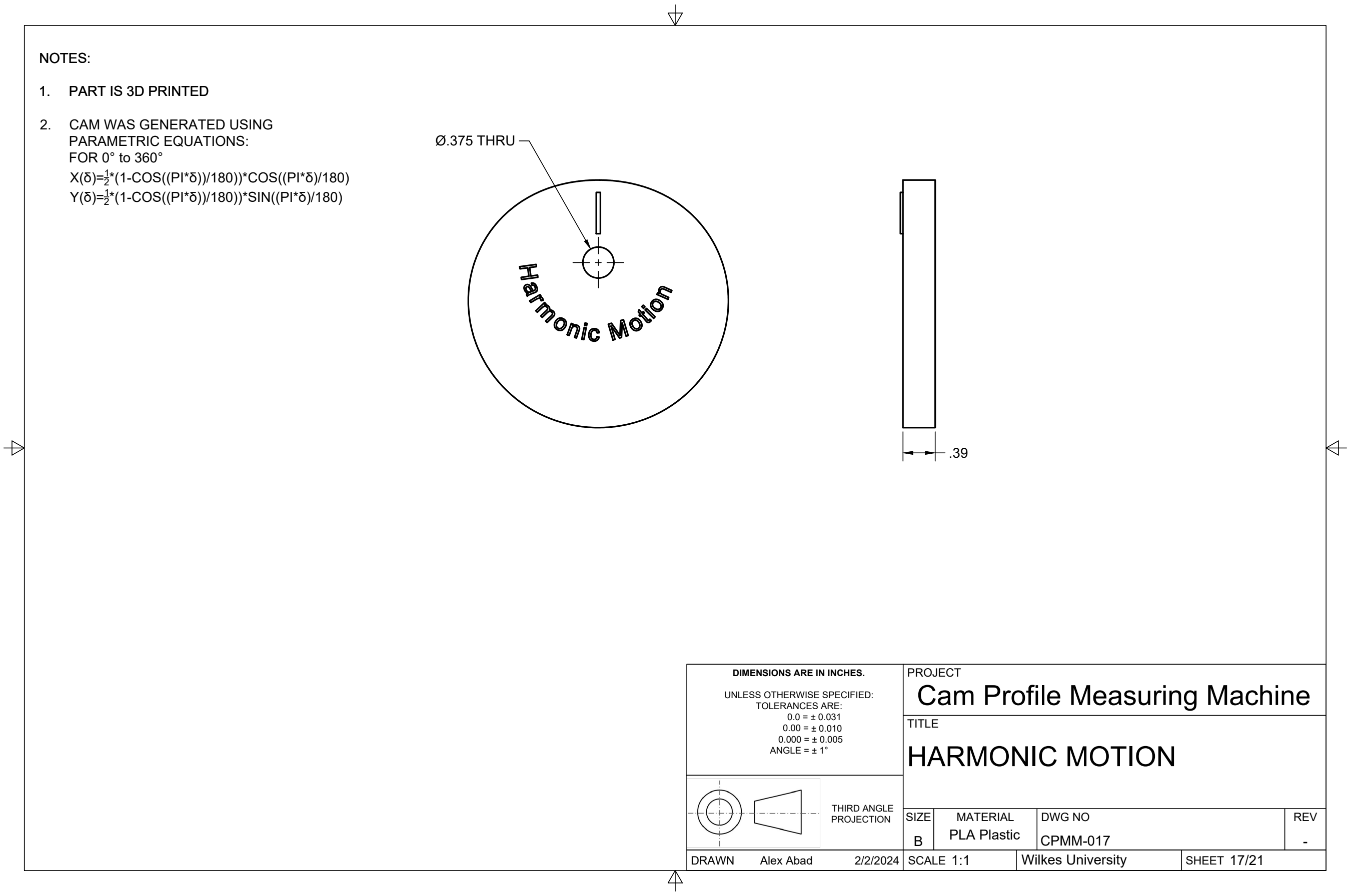
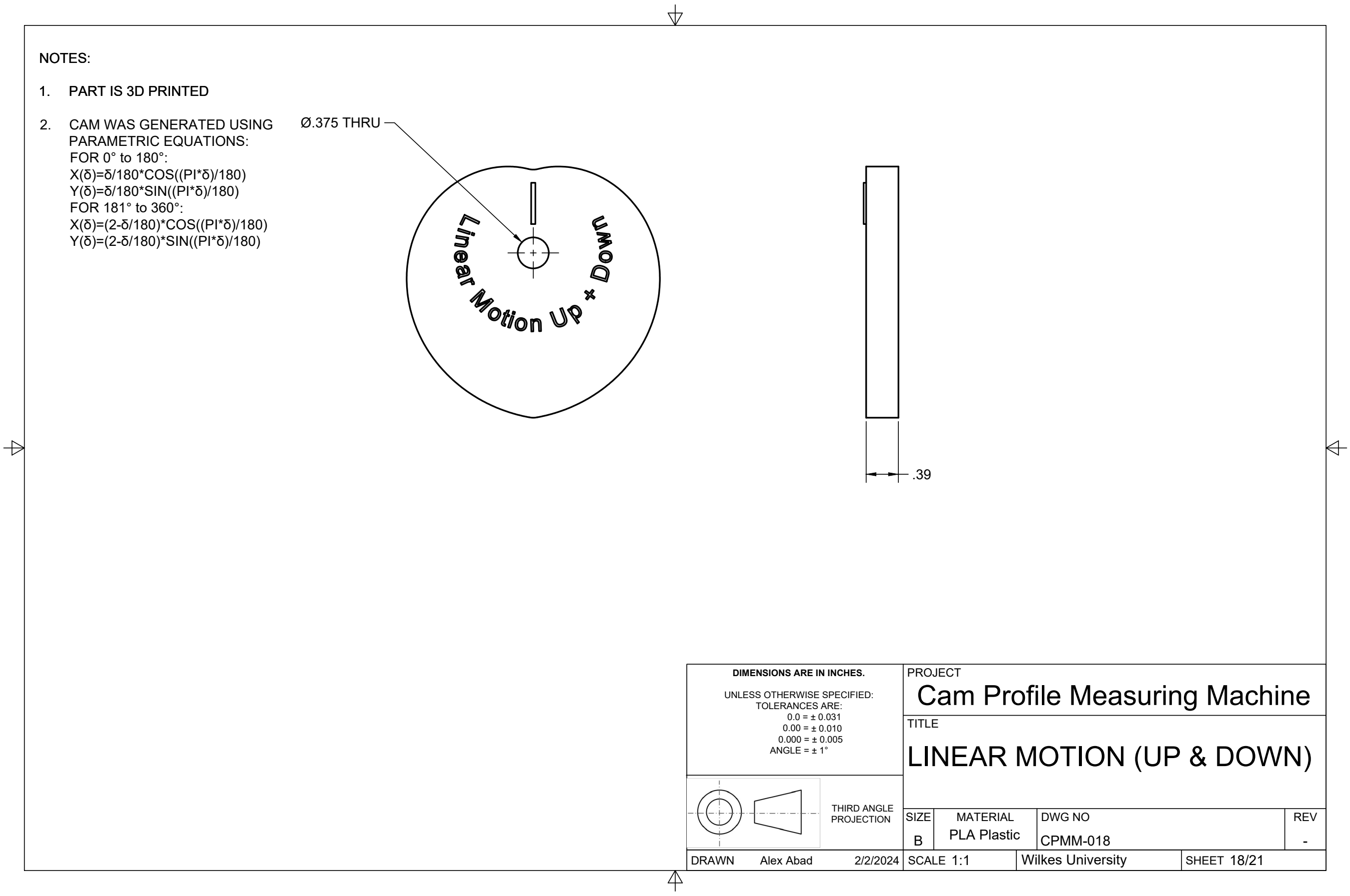
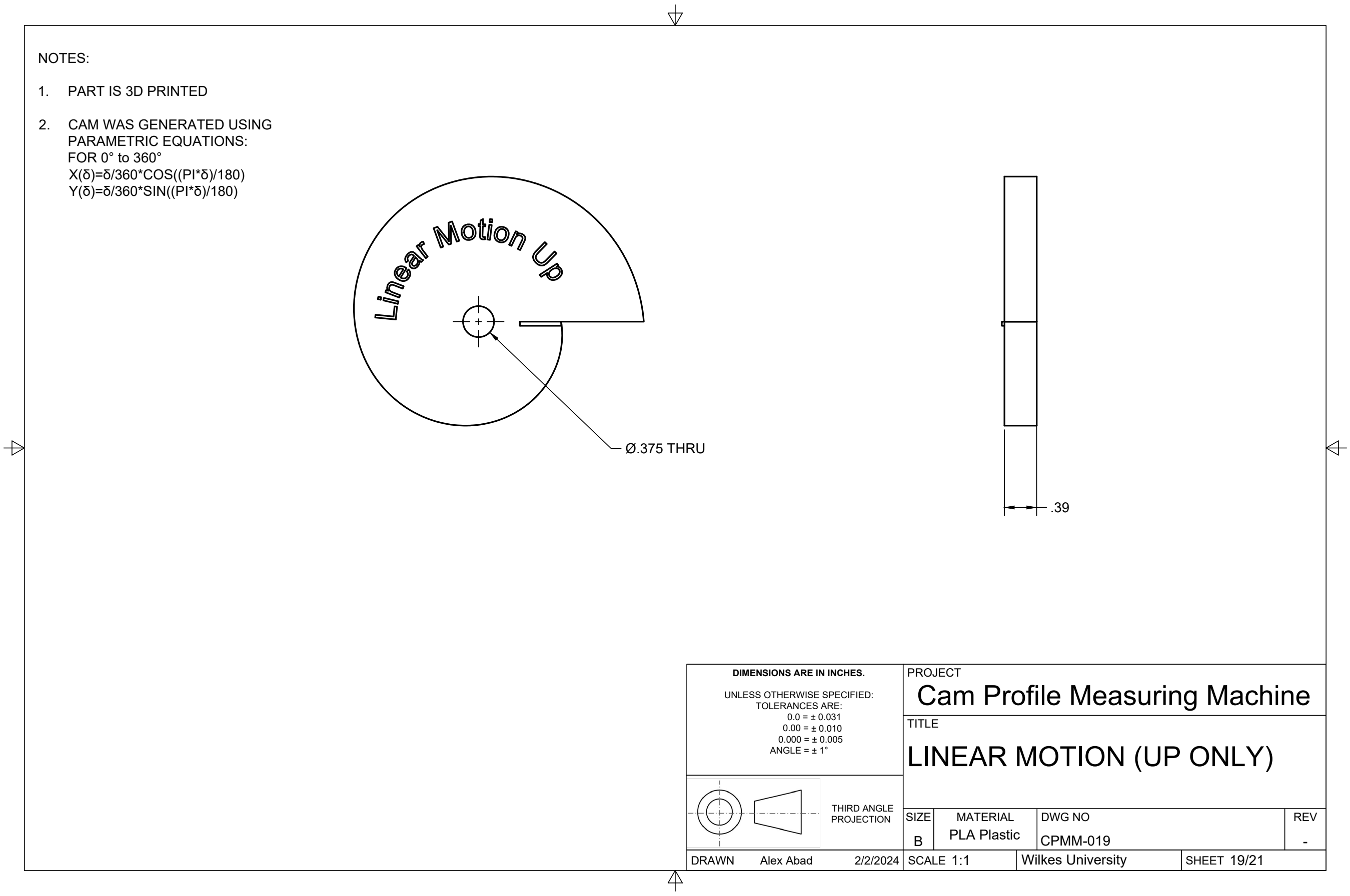
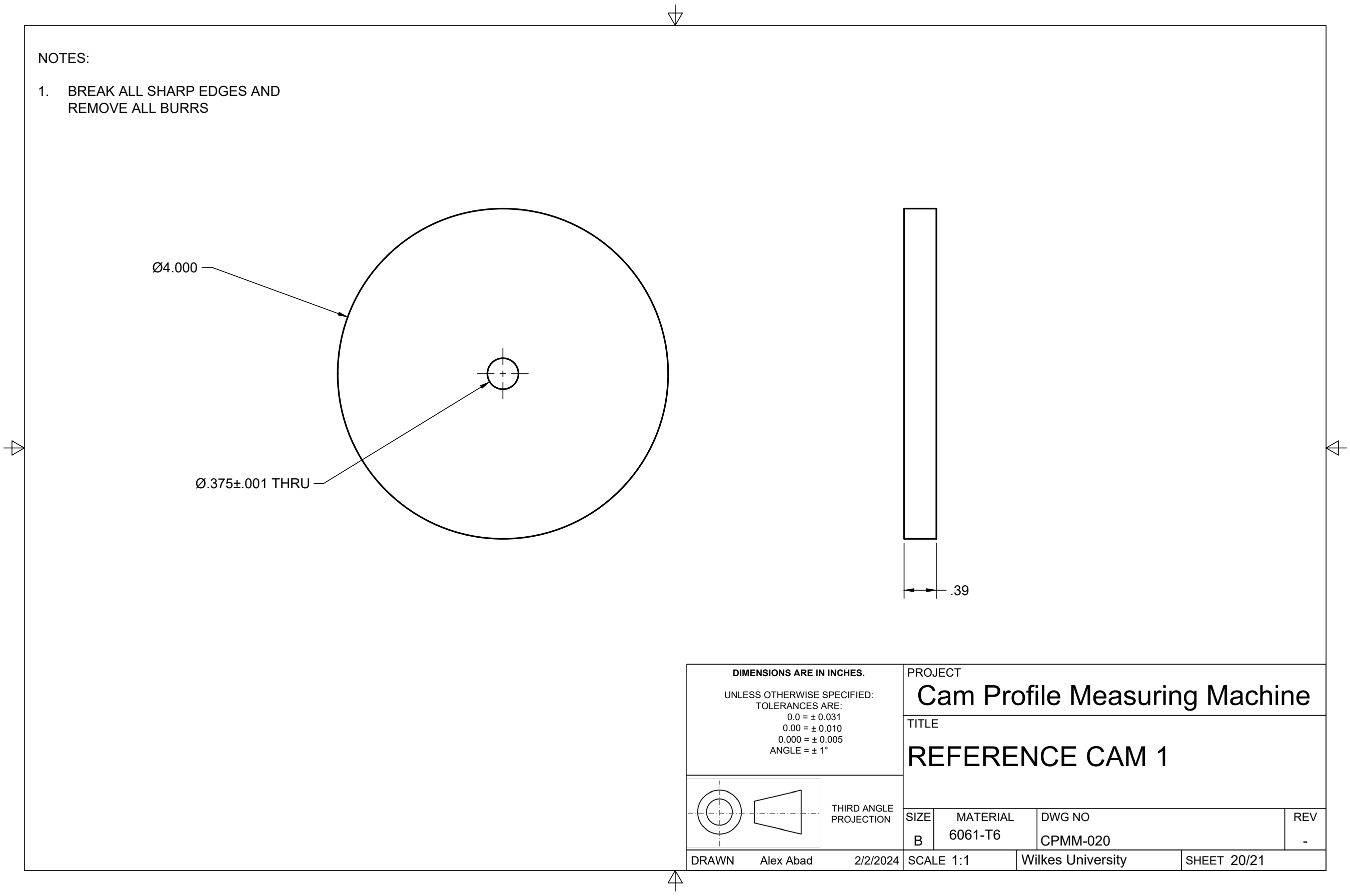
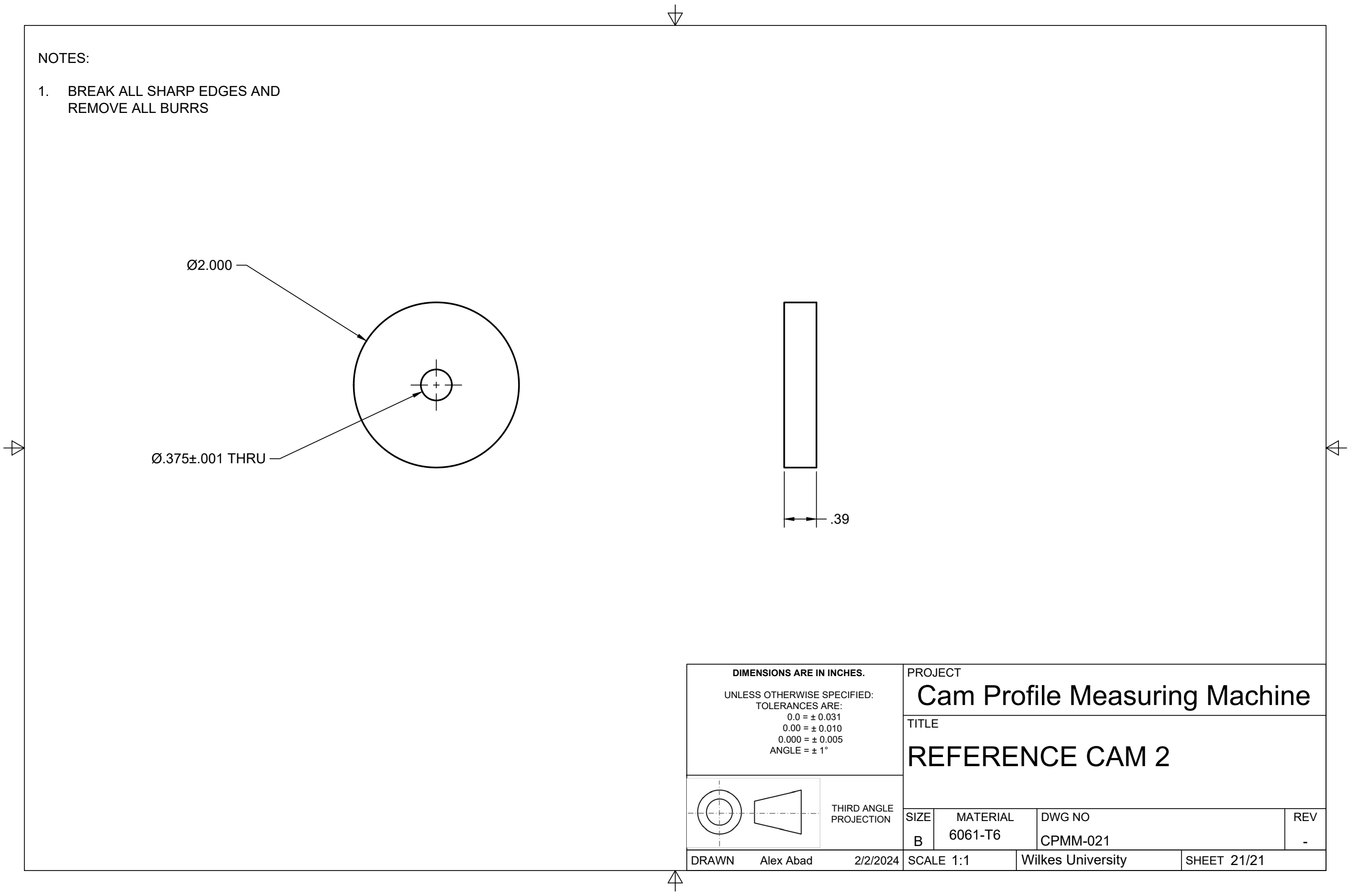
Y(δ)=1/90*(δ-90)*SIN((PI*δ)/180)
FOR 181° to 270°:
X(δ)=COS((PI*δ)/180)
Y(δ)=SIN((PI*δ)/180)
FOR 271° to 360°:
X(δ)=1/90*(-δ+360)*COS((PI*δ)/180)
Y(δ)=1/90*(-δ+360)*SIN((PI*δ)/180)
Ø.375 THRU
Double Dwell
.39
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
DOUBLE DWELL
SIZE B
MATERIAL PLA Plastic
DWG NO CPMM-016
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:1
Wilkes University
SHEET 16/21


NOTES:
1. PART IS 3D PRINTED
2. CAM WAS GENERATED USING
PARAMETRIC EQUATIONS:
FOR 0° to 360°
X(δ)=½*(1-COS((PI*δ))/180))*COS((PI*δ)/180)
Y(δ)=½*(1-COS((PI*δ))/180))*SIN((PI*δ)/180)
Ø.375 THRU
Harmonic Motion
.39
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED:
TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
HARMONIC MOTION
SIZE
B
MATERIAL
PLA Plastic
DWG NO
CPMM-017
REV
-
DRAWN
Alex Abad
2/2/2024
SCALE 1:1
Wilkes University
SHEET 17/21


NOTES:
1. PART IS 3D PRINTED
2. CAM WAS GENERATED USING
PARAMETRIC EQUATIONS:
FOR 0° to 180°:
X(δ)=δ/180*COS((PI*δ)/180)
Y(δ)=δ/180*SIN((PI*δ)/180)
FOR 181° to 360°:
X(δ)=(2-δ/180)*COS((PI*δ)/180)
Y(δ)=(2-δ/180)*SIN((PI*δ)/180)
Ø.375 THRU
Linear Motion Up + Down
.39
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED:
TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
LINEAR MOTION (UP & DOWN)
SIZE
B
MATERIAL
PLA Plastic
DWG NO
CPMM-018
REV
-
DRAWN
Alex Abad
2/2/2024
SCALE 1:1
Wilkes University
SHEET 18/21


NOTES:
1. PART IS 3D PRINTED
2. CAM WAS GENERATED USING PARAMETRIC EQUATIONS:
FOR 0° to 360°
X(δ)=δ/360*COS((PI*δ)/180)
Y(δ)=δ/360*SIN((PI*δ)/180)
Linear Motion Up
Ø.375 THRU
.39
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
LINEAR MOTION (UP ONLY)
SIZE B
MATERIAL PLA Plastic
DWG NO CPMM-019
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:1
Wilkes University
SHEET 19/21


NOTES:
1. BREAK ALL SHARP EDGES AND REMOVE ALL BURRS
Ø4.000
Ø.375±.001 THRU
.39
DIMENSIONS ARE IN INCHES.
UNLESS OTHERWISE SPECIFIED: TOLERANCES ARE:
0.0 = ± 0.031
0.00 = ± 0.010
0.000 = ± 0.005
ANGLE = ± 1°
THIRD ANGLE PROJECTION
PROJECT
Cam Profile Measuring Machine
TITLE
REFERENCE CAM 1
SIZE B
MATERIAL 6061-T6
DWG NO CPMM-020
REV -
DRAWN Alex Abad 2/2/2024
SCALE 1:1
Wilkes University
SHEET 20/21



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